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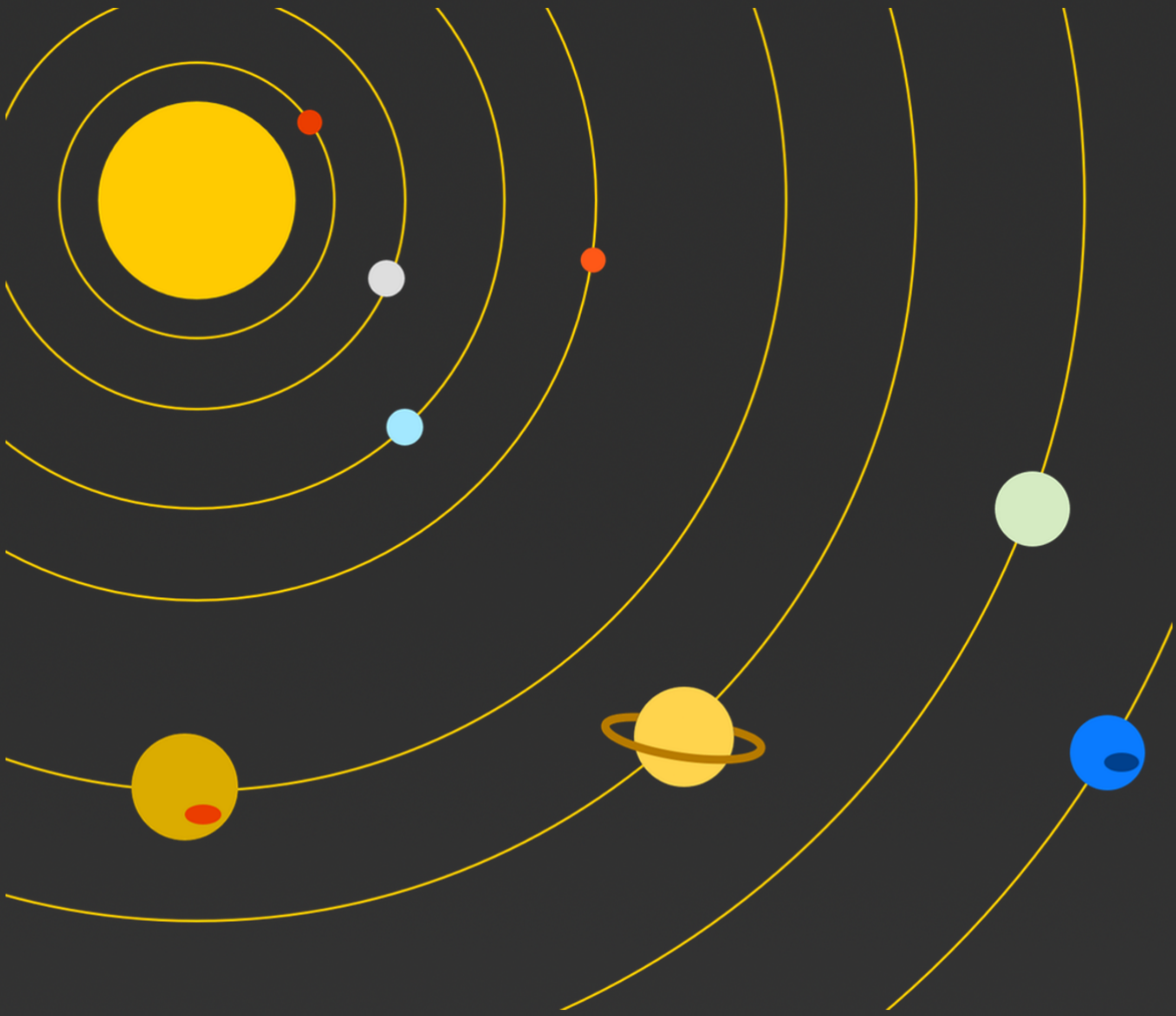

A2 PHYSICS

Fields, Motion and Thermal Physics

Stanley's Study Guides

Classroom Technologies

A2 Physics

Fields, Motion and Thermal Physics

written and illustrated by

Stanley Sim

Classroom Technologies

Contents

Preface

A-level Physics is a hard subject. It is hard because there is a lot of contents and the problems are hard to solve. In this series of Stanley's Study Guides, I attempt to summarise the concepts found in the Cambridge International Examination's A-level Physics syllabus (code 9702). This is to allow students to focus on the key concepts and prepare well for their examinations.

This volume on *Fields, Motion and Thermal Physics* covers the following topics of the A2-level Physics curriculum in the Cambridge International Examinations:

- Circular motion
- Gravitational fields
- Electric fields
- Simple harmonic motion
- Ideal gas
- Temperature
- Thermal properties of matter

All the best to your study.

Thank you
Stanley
June 2018

PS: To prevent conflict of interests, current students of Stanley Sim may always request for free copies of this eBook at classrmtech@gmail.com.

1

Circular motion

1.1 Radian

Definition of radian:

> *One radian is defined as the angle subtended at the centre of a circle by an arc length equal to the radius.*

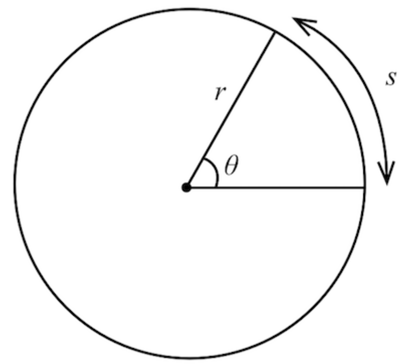

Fig. 1.1 Definition of 1 radian

The angle θ is 1 radian when the arc length s equals to the radius r.

1.2 Angular displacement

Definition of angular displacement:

> *The angle which an object moves around a circle is called the* ***angular displacement****.*

This angle is usually expressed in radians.

$$\pi = 180°$$

There are 2π radians in one complete circle.

The relationship between the angle in radian and the arc length of the circle is

$$\theta = \frac{s}{r} \tag{1.1}$$

where s is the arc length and r is the radius of the circle. θ is the angular displacement.

1.3 Angular speed and angular velocity

Definition of angular speed:

> ***Angular speed*** *is the rate of change of angular displacement.*

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad (1.2)$$

where ω is the angular speed. Since an object moves one circumference in one period time, we have

$$\omega = \frac{2\pi}{T} \qquad (1.3)$$

where T is the period of circular motion.

Linear speed can be obtained by

$$v = \frac{2\pi r}{T} \qquad (1.4)$$

The unit of angular speed is rad s^{-1} while the unit of linear speed is m s^{-1}.

Combining both equations, we have

$$v = r\omega \qquad (1.5)$$

1.4 Centripetal force

When an object moves in a circular motion, the velocity is always at a direction tangential to the circle. The velocity is always perpendicular to the direction of the centripetal force.

The **centripetal force** is the force that make an object moves in a circle. This centripetal force is a resultant force. It can originate from gravity for the case of the Moon revolving round the Earth, or electrostatic force for the case of electron moving round the nucleus. The centripetal force causing a car to move in a circle on the road is friction.

When the object is moving at a uniform speed in a circle, the centripetal acceleration and the centripetal force is always towards the centre of the circle and they have a constant magnitude.

Since acceleration is a consequence of a resultant force, the acceleration is always in the same direction as the centripetal force i.e. acceleration is always towards the centre of the circular motion.

The centripetal acceleration can be obtained from the angular speed by

$$a = r\omega^2 \tag{1.6}$$

Since $v = r\omega$,

$$\begin{aligned} a &= r\left(\frac{v}{r}\right)^2 \\ &= \frac{v^2}{r} \end{aligned} \tag{1.7}$$

By Newton's second law $F = ma$,

$$F = mr\omega^2 = \frac{mv^2}{r} \tag{1.8}$$

where m is is mass of the object moving in the circle and F is the centripetal force.

2
Gravitational Fields

2.1 Concept of a Gravitational Field and Force

A mass produces a gravitational field.

> *A gravitational field is a region in which a mass experiences a force.*

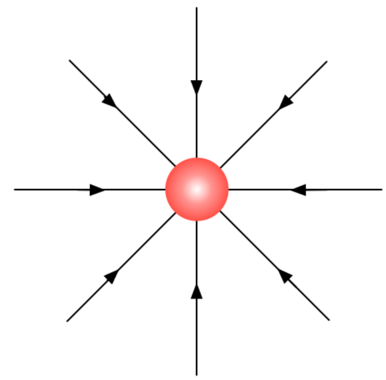

FIG. 2.1 GRAVITATIONAL FIELD
The direction of gravitational field is towards the mass that produces the field.

A mass that is present inside a **gravitational field** experiences a **gravitational force**. This gravitational field is produced by another mass.

2.2 Gravitational force

Definition of gravitational force:

> *The **gravitational force** between two masses is directly proportional to the product of the two masses and indirectly proportional to the square of the separation.*

$$F = -G\frac{Mm}{r^2} \qquad (2.1)$$

M is the mass that produces the gravitational field while m is the mass of the object in the gravitational field produced by M. G is the universal gravitational constant and has a value of 6.67×10^{-11} m^3 kg^{-1} s^{-2}. Negative denotes attraction. However, since gravity is always attractive, we may omit it during calculation.

The separation r is the distance between the centre of mass of the two masses. We often assume that the two masses are point masses if the separation is large relative to the radii of the masses. If the separation is not large, then it is important to use the distance between the centre of mass of the two masses. We should not use the separation between the two surfaces of the masses. Furthermore, one may safely assume that for a uniform sphere, the centre of mass is the centre of the sphere.

There are two regions about the field produced by a mass: the region outside the mass and the region inside. The region outside the mass follows the inverse square law relationship of $\frac{1}{r^2}$.

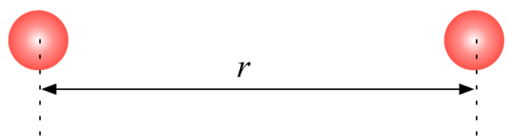

FIG. 2.2 DEFINITION OF DISTANCE IN THE UNIVERSAL LAW OF GRAVITATION
The distance *r* to calculate the gravitational force between two objects stretches from the centre of masses of the two object.

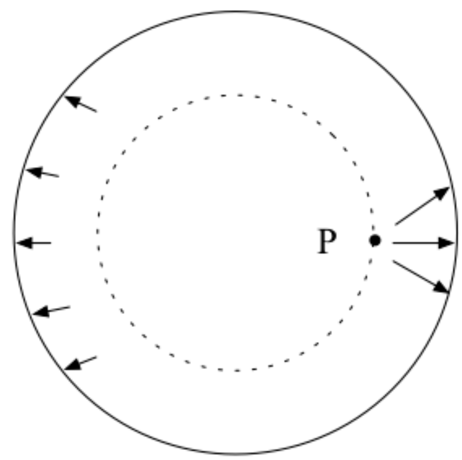

FIG. 2.3 GRAVITATIONAL FIELD INSIDE EARTH
Inside the mass, the force is directly proportional to the distance from the centre of the mass. This is because as you proceed nearer to the centre of the mass, there is less mass "below" you. The part of the mass "above" you pulling you "up" is offset by the mass "below" you pulling you down.

For a uniform sphere, the mass of the sphere can be considered to be a point mass at its centre. This point is its centre of mass. The law of gravitation is only applicable for region outside the mass.

2.3 Gravitational Field Strength

Definition of gravitational field strength at a point:

> ***Gravitational field strength*** *at a point is the gravitational force acting on a unit mass at that point.*

From the definition,

$$\begin{aligned} \text{Force} &= mg \\ mg &= G\frac{Mm}{r^2} \\ g &= G\frac{M}{r^2} \qquad (2.2) \end{aligned}$$

We can see that gravitational field strength only depends on the gravity-producing mass and the distance from it. It is not dependent on the test mass.

There is only one value of gravitational field strength at any particular point. If there are multiple masses creating gravitational fields, the gravitational field strength at any particular point would be the vector sum of all the field strengths due to the the different masses.

2.4 Gravitational force and centripetal force

When a mass orbits around a planet, the centripetal force that makes the mass moves in a circular motion is provided by the gravitational force of the planet on the mass.

$$\begin{aligned} G\frac{Mm}{r^2} &= \frac{mv^2}{r} \\ GM &= rv^2 \qquad (2.3) \end{aligned}$$

It is important to understand that **(2.3)** only applies if the mass that produces the gravitational field(not the orbiting mass) is much larger than the orbiting mass.

Substituting $v = \dfrac{2\pi r}{T}$ into **(2.3)** , we have

$$\begin{aligned} GM &= r\left(\frac{4\pi^2 r^2}{T^2}\right) \\ T^2 &= \frac{4\pi^2}{GM}r^3 \qquad (2.4) \end{aligned}$$

This equation tells us that $T^2 \propto R^3$. This relationship is also known as the Kepler's third law of planetary motion. This relationship also tell us that the square of the period of revolution is directly proportional to the cube of the radius of orbit, and is independent of the orbiting mass.

2.5 Geostationary orbits

Satellites are put into orbits to provide certain functions. Sometimes, these functions may require the satellites to be at fixed locations at all times in the sky. To achieve this, these satellites are put into **geostationary orbits** and they are called geostationary satellites.

> *A geostationary orbit is an orbit such that the satellite has a period identical to the rate of rotation of the Earth.*

Since these satellites have the same period of orbit as the Earth's period of rotation i.e. 24 hours, the radius r would have a specific value.

$$
\begin{aligned}
r^3 &= \left(\frac{GMT^2}{4\pi^2}\right) \\
&= \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{4\pi^2} \times (24 \times 3600)^2 \\
r &= 4.23 \times 10^7 \text{ m}
\end{aligned}
$$

Subtracting from Earth's radius, the distance of a geostationary satellite from Earth's surface is

$$
\begin{aligned}
r_{\text{surface}} &= 4.23 \times 10^7 - 6.4 \times 10^6 \\
&= 3.59 \times 10^7 \text{ m}
\end{aligned}
$$

2.6 Gravitational field strength on the surface of the Earth

When we calculate weight of an object, we always use the formula $w = mg$.
g is referred to as the gravitational field strength (although it is commonly stated as the gravitational acceleration). Near the Earth's surface, the field strength of the Earth is

$$
\begin{aligned}
g_{\text{surface}} &= G\frac{M}{r_{\text{radius}}^2} \\
&= G\frac{M}{(6400 \times 10^3)^2}
\end{aligned}
$$

The field strength at 10 km above the surface would be

$$g_{10\text{ km}} = G\frac{M}{(6410 \times 10^3)^2}$$

The difference in the gravitational field strengths is

$$\begin{aligned}\Delta g &= GM\left(\frac{1}{(6400 \times 10^3)^2} - \frac{1}{(6410 \times 10^3)^2}\right)\\ &= 7.6 \times 10^{-17}\ \text{N kg}^{-1}\end{aligned}$$

The difference among them is negligible. Hence we assume that the gravitational field strength on Earth's surface is constant.

2.7 Gravitational potential and energy

Definition of gravitational potential at a point:

> *Gravitational potential at a point is the amount of work needed to bring a unit mass from infinity to that point.*

Gravitational potential is negative because gravity is attractive in nature. The gravitational potential at infinity is defined to be zero. A negative potential means that no external work is needed to bring a unit mass from infinity to that point, since the gravity-producing mass would be doing the work to pull the unit mass to that point. It also mean the any mass placed at that point will be bounded by gravity to the field-producing mass.

Mathematically,

$$\phi = -\frac{GM}{r} \tag{2.5}$$

where ϕ is the gravitational potential at a point r from the mass M.

Note that potential is a scalar quantity. Hence, the potential at a point is simply the algebraic sum of the potential of the different masses at that point.

$$\phi_{\text{total}} = -\frac{GM_1}{r} - \frac{GM_2}{r} - \ldots \tag{2.6}$$

Similarly, gravitational potential energy is defined as

Gravitational potential energy of a mass at a point is the amount of work needed to bring the mass from infinity to that point.

Mathematically,

$$E_{\text{P}} = -G\frac{Mm}{r} \tag{2.7}$$

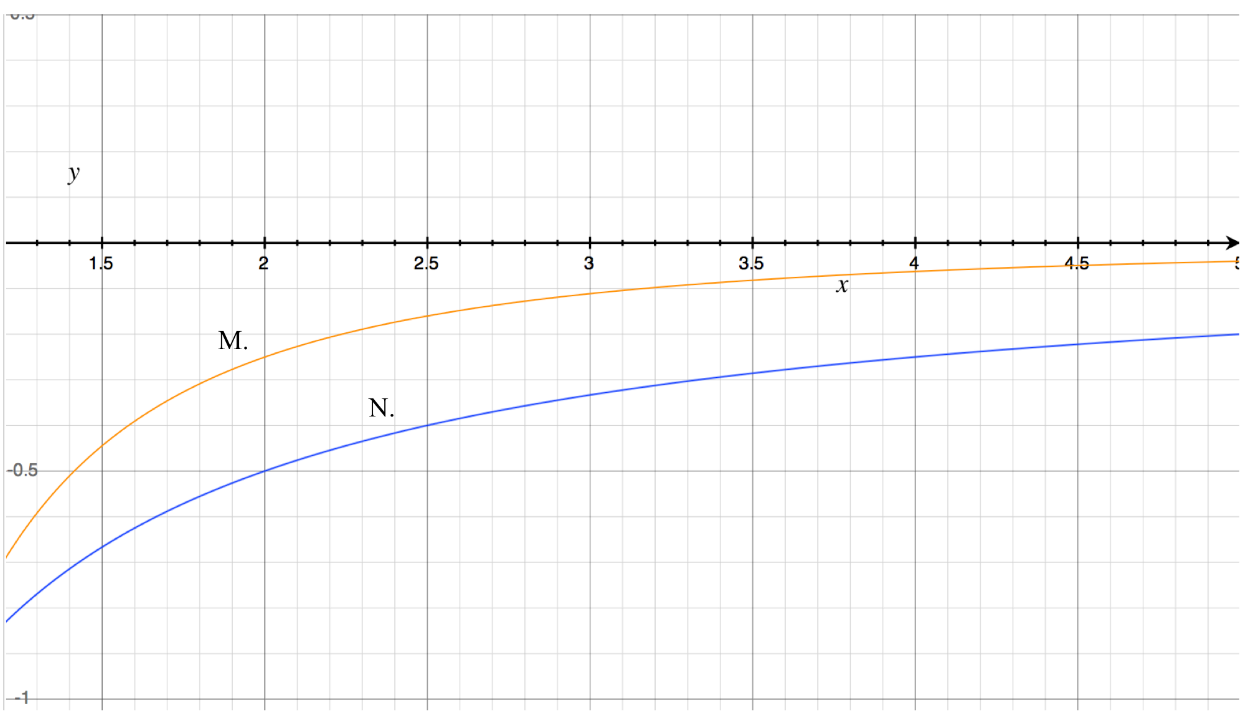

FIG. 2.3 COMPARISON BETWEEN GRAVITATIONAL FIELD AND POTENTIAL

The orange graph represents the gravitational field while the blue graph represents the gravitational potential.

The gravitational field follows the relationship while potential follows the relationship. As such, the gravitational field has a value nearer to the horizontal axis than the potential graph.

It is also important to note that since gravitational field and potential are negative, they are always below the horizontal axis.

Both the gravitational field and potential will be zero at infinity.

You may find this concept similar to gravitational force and gravitational field strength. Both gravitational force and potential energy involves the product of two masses while field strength and potential involves just the gravity-producing mass M without the test mass m.

2.8 Field and potential near the Earth's surface

Gravitational potential energy is often quoted with the formula

$$E = mgh \qquad (2.8)$$

This formula assumes that the change in height is insignificant compare to the radius of the Earth. This formula calculates the change in the potential energy due to a change in position.

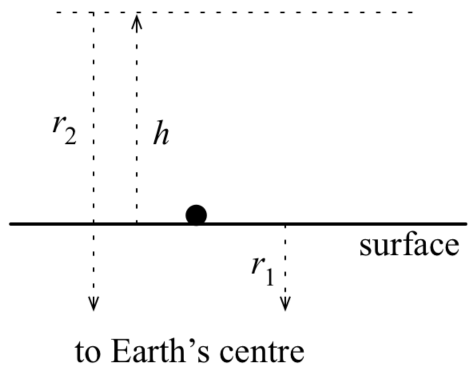

Fig. 2.4 Gravitational potential energy near Earth's surface.

The formula $E_P = -G\dfrac{Mm}{r}$ calculates the actual amount of potential energy a mass possess due to its position at a distance r from the mass M. This formula does not calculate the change in potential energy. To calculate the change in potential energy in **Fig. 2.4**,

$$\begin{aligned}\Delta E_P &= -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right) \qquad (2.9)\\ &= GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\\ &= GMm\left(\frac{r_2 - r_1}{r_1 r_2}\right)\\ &= gm(r_2 - r_1)\\ &= mgh\end{aligned}$$

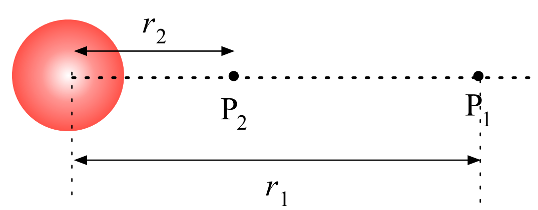

Fig. 2.5 Gravitational potentials of two points from a mass

To calculate the change in potential energy of a mass moved from P_1 to P_2 that are NOT near to each other, Eq (2.9) need to be used instead of Eq (2.8)

Here, we assume that since $r_2 - r_1$is small, $r_1 r_2 \approx r_1^2$.

Hence,

$$\frac{GM}{r_1 r_2} \approx \frac{GM}{r_1^2}$$
$$= g$$

Near the Earth's surface, the gravitational field may be assumed to be uniform. This allows us to use $E_P = mgh$ to calculate change in potential energy and g (= 9.81 N kg^{-1}) as the gravitational field strength on different points to calculate the weight on an object.

3
Electric fields

3.1 Electric force

The law of electrostatics states that

> *The electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between the point charges.*

Mathematically,

$$F = \pm \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r^2} \tag{3.1}$$

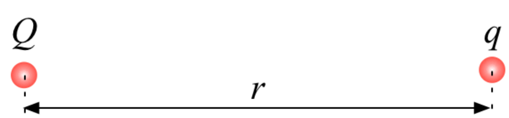

FIG. 3.1 COULOMB'S LAW OF ELECTROSTATICS

Coulomb's law is applicable to point charges. If the charge is an extended object, then the distance between the charges should be from the centre of one charge to the centre of the other charge.

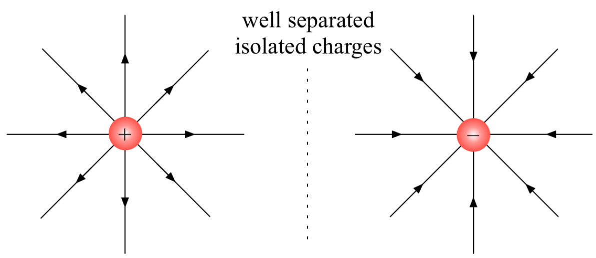

FIG. 3.2A ELECTRIC FIELD PATTERNS OF SOME CHARGE CONFIGURATIONS

Isolated charges are charges where their fields are not affected by other charges. Electric fields of isolated charges are radial in direction. Electric fields line always start from a positive charge and ends at a negative charge.

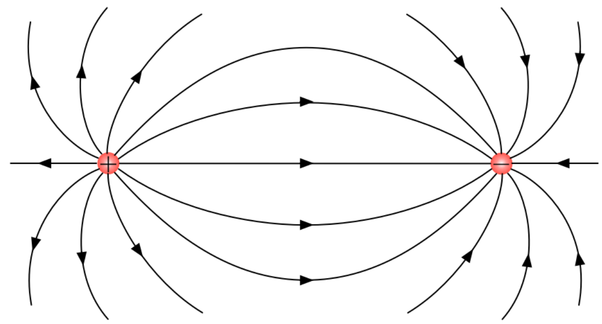

FIG. 3.2B ELECTRIC FIELD PATTERN OF TWO UNLIKE CHARGES

Electric field patterns of two unlike charges. Observe that electric field lines do not cross each other's.

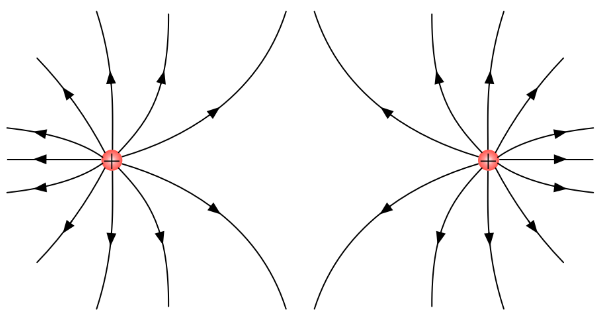

FIG. 3.2C ELECTRIC FIELD PATTERN OF TWO LIKE CHARGES

The field lines do not cross. Note that at the centre is the point where there is no net electric field. A charge placed here will experience no force.

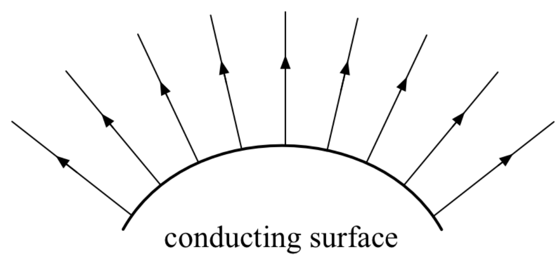

FIG.3.2D ELECTRIC FIELD OF A CONDUCTING SURFACE

Electric field lines is always perpendicular to the surface of a conducting material.

This is Coulomb's law. **Electric force** is sometimes also known as the Coulomb force. ε_0 is the permittivity of free space. It is a measure of how well vacuum can permit electric field lines. Its value is 8.85×10^{-12} F m^{-1}.

The plus-minus(±) is applicable in this equation because electric charges has positive and negative charges. Two like-charges repel while two unlike-charges attract. When the force is repulsion, + would be used while – is used for attractive force.

r is the distance between the two point charges. If the electric charge is a uniform conducting sphere, the charge may be considered to act from the centre of the sphere. The electric field of a point anywhere inside the conducting sphere ($d < r$) is zero and Coulomb's law is only applicable from the radius of the sphere and beyond ($d \geq r$).

3.2 Electric field of a point charge

Definition of an electric field:

The electric field is a region which a charge experiences an electric force.

A charge produces an **electric field**. A point in the electric field has a value called electric field strength. Electric field always start from a positive charge and end at a negative charge.

Electric field strength of a point is the electric force acting on a unit charge at that point.

From the definition,

$$E = \pm \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \tag{3.2}$$

where E is the electric field strength.

3.3 Electric potential

Definition of electric potential at a point:

Electric potential at a point is the work done to bring a unit positive charge from infinity to the point.

The **electric potential** of a point at infinity is 0 J C^{-1}. To bring a unit positive charge to a point at a distance r from the charge that produces the field, external energy is required. This energy is transferred to the unit positive charge and it will have an amount of energy equal to $\phi = +\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$. A unit negative charge placed at infinity will be attracted by the field-producing charge and moved to the point r away from the field-producing charge. The energy it lost equal to $\phi = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$.

Hence, the potential of a point at a distance r away from the field-producing charge is given by the formula

$$\phi = \pm\frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \tag{3.3}$$

3.4 Relationship between field strength and potential

The magnitude of the electric field strength of a point equals to the rate of change of electric potential with respect to distance from the field-producing charge. The field strength of a point is the negative of the potential gradient at that point.

The field strength has an opposite sign to the potential gradient because the potential gradient increases in the negative direction(sloping steeper) as the unit charge moves nearer to the field-producing charge.

Unit charge	The specific electric charge
Electric field strength	Electric force
Electric potential	Electric potential energy

Table 3.1 Field strength vs force and potential vs potential energy

Both field and potential reference to specific points in space while force and potential energy are quantities which a charge possesses due to its position in an electric field.

3.5 Electric potential energy

Definition of electric potential energy:

Electric potential energy of a charge at a point is the amount of work done to bring the charge from infinity to that point.

Mathematically, the electric potential energy of a charge q is

$$\begin{aligned} E_\mathrm{P} &= q\phi \\ &= \pm\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r} \end{aligned} \tag{3.4}$$

3.6 Comparison between electric fields and gravitational fields

	Gravitation	Electric
Field and force	Always negative. Field lines are always towards the mass. $g = -G\dfrac{M}{r^2}$ $F = -G\dfrac{Mm}{r^2}$	Can be positive or negative. Direction of field lines depends on the charge. Field lines points away from a positive charge but points towards a negative charge. Direction of force depends on the charge in the field. The force on positive charge is in the same direction as the field lines but the force on a negative charge is opposite to the direction of the field lines. $E = \pm\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$ $F_E = \pm\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{r^2}$
Potential and potential energy	Always negative. Negative represents attraction and gravity is always attractive. $\phi = -G\dfrac{M}{r}$ $E_P = -G\dfrac{Mm}{r}$	Potential produced by a positive charge is positive but is negative when it is produced by a negative charge. Potential energy depends force acting on the charge. If the charge is attracted to charge Q, it has a negative potential energy. If it is repelled by charge Q, it has positive potential energy. $\phi = \pm\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r}$ $E_P = \pm\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{r}$

TABLE 3.2 COMPARISON BETWEEN GRAVITATION AND ELECTROSTATICS.

4 Simple harmonic motion

4.1 Simple harmonic motion

An object moving in an **SHM** must obey the relation

$$a = -\omega^2 x \tag{4.1}$$

where a is the acceleration of the object, x is the displacement and ω is the **angular frequency** of the oscillator. ω is related to the frequency f of the oscillator by the relation $\omega = 2\pi f$.

A body is moving in SHM if its acceleration is directly proportional to the displacement and in the opposite direction to the displacement.

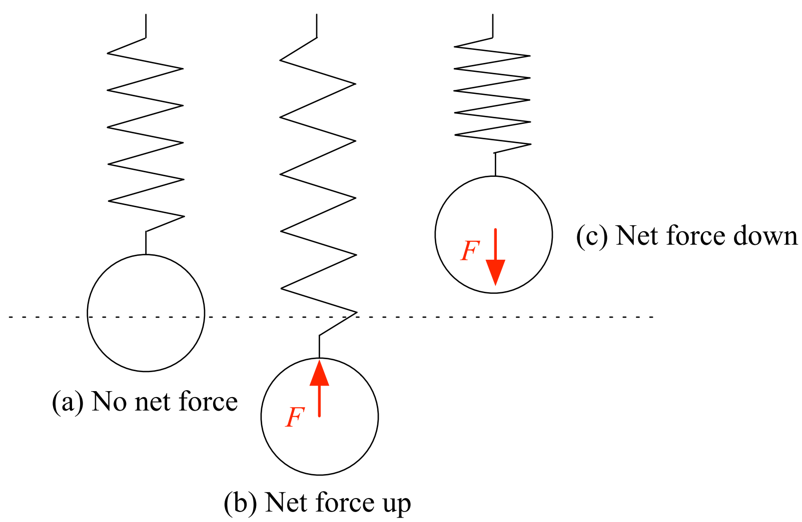

FIG. 4.1 NET FORCE ACTING ON A SHM OSCILLATOR
The acceleration of an oscillator is always towards equilibrium. When the oscillator is displaced down, the net force is pointing up. It is important to note that the net force is the sum of the weight of the object and the spring force on the object.

In **Eq (4.2)**, we observe the following:

- the acceleration of the oscillator is always opposite to the displacement,
- the magnitude of the acceleration is directly proportional to the displacement.

From these two points, we see that the acceleration is greatest when the oscillator is at the maximum displacement while it is zero when the oscillator is at equilibrium. Since force and acceleration is directly proportional, we see that the restoring force of the oscillator is correspondingly largest at the maximum displacement and zero at equilibrium.

4.2 SHM in a spring

When a object hung on a spring oscillates, it is moving in SHM. There is a restoring force from the spring causing the object to move up and down. There are two ways to start the motion:

1. displace the object to a certain distance and release, or
2. with the oscillator at equilibrium, give it a push such that it starts moving.

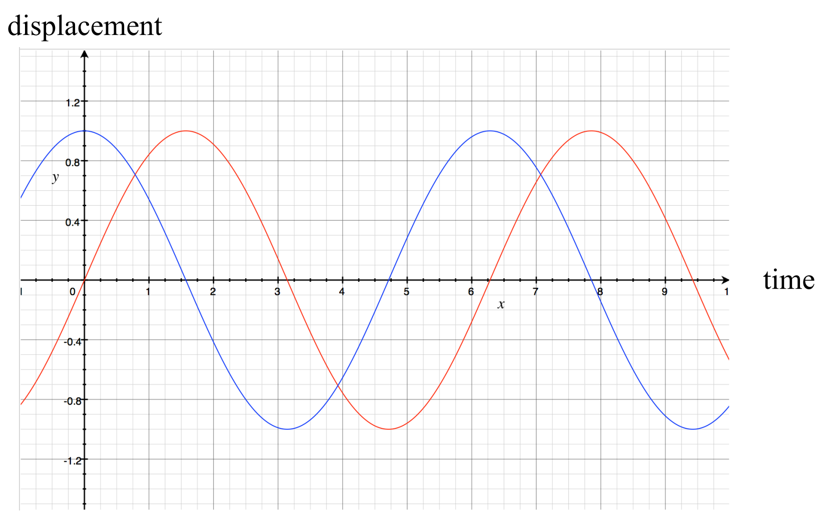

Fig. 4.2 Two equations to describe an SHM oscillator
An oscillator that is given a push at equilibrium follows the red graph while an oscillator being displaced to amplitude at t = 0 follows the blue graph

The red graph would have an equation of

$$x = x_0 \sin \omega t \tag{4.2}$$

while the blue graph is $x = x_0 \cos \omega t$.

Since velocity is the derivative of displacement with time,

$$\begin{aligned} v &= \frac{dv}{dt} = x_0 \omega \cos \omega t \\ &= v_0 \cos \omega t \end{aligned} \tag{4.3}$$

Acceleration is the derivative of velocity with time, hence

$$\begin{aligned} a &= \frac{dv}{dt} = -x_0 \omega^2 \sin \omega t \\ &= a_0 \sin \omega t \end{aligned} \tag{4.4}$$

Combining with **Eq (4.2)** , we have

$$a = -\omega^2 x$$

Equations review

The equations below relate the quantities of SHM(displacement and velocity) to time.

$$\begin{aligned} x &= x_0 \sin \omega t \\ v &= x_0 \omega \cos \omega t \\ a &= -x_0 \omega^2 \sin \omega t \end{aligned}$$

The equations below relate velocity and acceleration to displacement

$$\begin{aligned} v &= \pm \omega \sqrt{x_0^2 - x^2} \\ a &= -\omega^2 x \end{aligned} \tag{4.5}$$

4.3 Energy of SHM

In an SHM, the total mechanical energy is always equal. The oscillator moves at the highest speed at the point of equilibrium. Thus it has the largest kinetic energy at the point of equilibrium ($x = 0$). At the maximum amplitude, the speed is zero. Hence, the potential energy is largest at the maximum amplitude. At anywhere between the equilibrium and maximum displacement, the sum of the potential energy and the kinetic energy is the total mechanical energy.

With the velocity formula, you can derive the kinetic energy equation for an oscillator.

$$\begin{aligned}\text{kinetic energy} &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}m\omega^2(x_0^2 - x^2) \end{aligned} \tag{4.5}$$

The maximum KE can be found when $x = 0$:

$$\text{KE}_{\text{max}} = \frac{1}{2}m\omega^2 x_0^2 \tag{4.6}$$

In SHM,

$$\text{total mechanical energy} = \text{kinetic energy} + \text{potential energy}$$

Hence,

$$\begin{aligned}\text{total mechanical energy} &= \text{maximum kinetic energy} \\ \text{total mechanical energy} &= \frac{1}{2}m\omega^2 x_0^2 \\ \text{potential energy} &= \frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2(x_0^2 - x^2) \\ &= \frac{1}{2}m\omega^2 x^2 \end{aligned} \tag{4.7}$$

Equations review

$$\text{total mechanical energy} = \frac{1}{2} m x_0^2$$
$$\text{potential energy} = \frac{1}{2} m x^2$$
$$\text{kinetic energy} = \frac{1}{2} m (x_0^2 - x^2)$$

4.4 Phases of SHM

From the equations of SHM,

$$x = x_0 \sin \omega t$$
$$v = x_0 \omega \cos \omega t$$
$$a = -x_0 \omega^2 \sin \omega t$$

we can see that x and v is $\frac{\pi}{2}$ out of phase while x and a is π out of phase.

v and a is also $\frac{\pi}{2}$ out of phase.

4.5 Damping in SHM

In a damped oscillation, energy is lost. The rate at which the energy lost follows an exponential relationship. An oscillator can be damped in three ways:

- light damping
- critical damping
- heavy damping

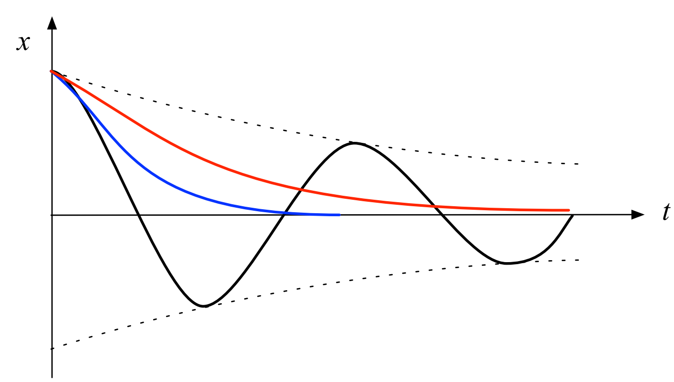

FIG. 4.3 LIGHT, CRITICAL AND HEAVY DAMPING
In light damping, the oscillator manages more than one complete oscillation. In critical damping, the oscillator reaches equilibrium in the shortest time. In heavy damping, the oscillator do not reach equilibrium.

In light damping, the oscillator can perform multiple complete oscillations while is amplitude decreases gradually at every oscillation.
In heavy damping, the oscillator could not reach the point of equilibrium.
In critical damping, the oscillator reach the point of equilibrium within the shortest time.

To create damping, find ways to reduce the energy of the oscillator.

1. ways to increase friction
2. ways to increase resistance(air or liquid)

As such, increasing the mass of a spring **WILL NOT** increase damping.

4.6 Resonance

Resonance occurs when the natural frequency of vibration of an object is equal to the driving frequency, giving a maximum amplitude of vibration.

Damping an oscillator will cause the natural frequency of the oscillator to decrease very slightly. It can be taken as having little effect to the natural frequency. The driving frequency to cause resonance would remains the same in a damped oscillator according to the graph.

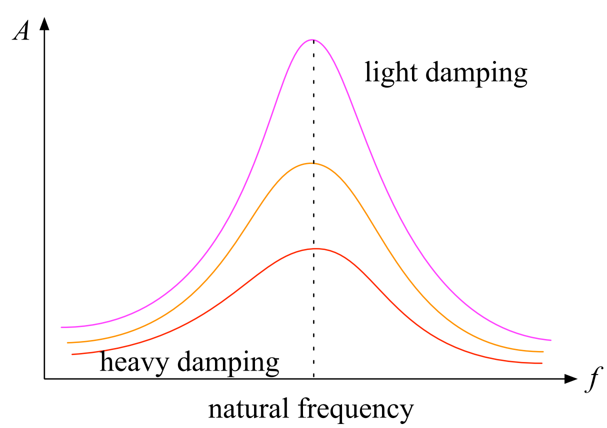

FIG. 4.4 EFFECT OF DAMPING TO THE NATURAL FREQUENCY OF OSCILLATOR

In a lightly damped oscillator, a driving frequency that equals to the natural frequency of the oscillator causes it to oscillate with a large amplitude.

A heavier-damped oscillator will produce a smaller amplitude but the natural frequency remains constant.

5
Ideal gas

5.1 Ideal gas

An **ideal gas** is a gas that satisfy the following equation

$$pV = nRT \tag{5.1}$$

where p is the pressure of the gas, V is the volume of the gas, n is the amount of gas in moles, R is the universal **molar gas constant** = 8.31 J mol^{-1} K^{-1}, and T is the temperature of the gas in kelvin.
One mole of gas contains 6.02×10^{23} particles, which is also known as the Avogadro's Number.

$$\begin{aligned} n &= \frac{N}{N_A} \\ pV &= \frac{N}{N_A}RT \\ &= N\frac{R}{N_A}T \\ pV &= NkT \end{aligned} \tag{5.2}$$

N is the number of particles and k is the Boltzmann constant.

$$k = \frac{R}{N_A} \tag{5.3}$$

5.2 Kinetic theory of gas

The **kinetic theory of gas** is a theory that links the microscopic properties of gas to the macroscopic measurable properties. An example of a microscopic property is speed of the particles. Macroscopic properties are pressure, volume and temperature. To link the two set of properties, some assumptions need to be made regarding the gas.

Basic assumptions of the kinetic theory of gases

1. A gas is made of a very large number of particles.
2. There are no forces between the particles except during collisions.
3. The volume of the particles is negligible as compared to the volume of the gas.
4. The collisions between particles is perfectly elastic.
5. The particles are moving at constant velocity except during collisions.

5.3 Brownian motion

Brownian motion is the observation that bright specks of light are moving in random zigzag motion. The smoke particles are seemingly changing directions without any external factors. Brownian motion can be explained by the kinetic theory of gas.

1. Air particles are moving at high speeds.
2. Collisions between the invisible air particles and the smoke particles cause the smoke particles to change their direction of motion.
3. The random motion of air molecules cause the random zigzag motion observed in the smoke particles.

5.4 Pressure of a gas

Air particles move at high speeds. When they collide with the walls of the container that the gas is kept, the particles bounce back. This results in a change in momentum of the air particles. The change in momentum of the air particles causes an equal but opposite force on the wall of the container. The force creates the pressure on the wall.

Relationship between pressure and speed of molecules

The pressure and volume is related to the average speed of the air particles in the gas.

$$P = \frac{1}{3}\frac{Nm < c^2 >}{V} \qquad (5.4)$$

$< c^2 >$ is the mean-square speed of the particles, N is the number of particles, V is the volume of the gas and P is the pressure of the gas and m is the mass of one particle.

Since density $= \dfrac{Nm}{V}$,

$$p = \frac{1}{3}\rho < c^2 > \tag{5.5}$$

where ρ is the density of the gas.

This equation tells you that the pressure of the gas depends only on the density of the gas and the mean-square speed of the gas particles.

5.5 Root-mean-square speed of particles

A gas is made of a large number of particles. The average speed of these particles obtained by summing up the individual speeds and dividing by the number of particles is NOT a good average because the particles are moving in a 3-dimensional space. A better average to represent the particles is the root-mean-square (r.m.s.) speed.

The root-mean-square speed is obtained by

1. Summing up the square speeds of the particles (square speeds),
2. Divide the sum by the number of particles (mean-square speed),
3. Square root the value from #2 (root-mean-square speed).

r.m.s. vs mean speed

5 particles have speed of unit 2, 2, 4, 6, 7. Calculate the mean and r.m.s. speeds of the particles.

$$\text{mean speed} = \frac{2+2+4+6+7}{5} = 4.2$$

$$\text{r.m.s. speed} = \sqrt{\frac{2^2+2^2+4^2+6^2+7^2}{5}} = 4.7$$

5.6 Kinetic energy of the gas particles

From (5.4),

$$PV = \frac{1}{3}Nm < c^2 > = nRT$$

$$\frac{1}{2}m < c^2 > = \frac{3nRT}{2N}$$

$$E_k = \tfrac{3}{2}kT \qquad (5.6)$$

This equation tells us that the mean average kinetic energy of a particle is directly proportional to the thermodynamic temperature of the gas.

The total internal energy of an ideal gas, which consists of only the kinetic energy, is

$$E_k = \tfrac{3}{2}NkT \qquad (5.7)$$

6

Temperature

6.1 Temperature and thermal energy

Definition of temperature:

> *Temperature is a measure of the degree of hotness.*

Temperature is not energy. An object may be hot but carries little heat energy while another object may be less hot but carries a lot of heat energy. A spark is hot but carries little heat energy. A litre of boiling water may has a lower temperature than the spark, but the water carries much more heat energy.

Thermal energy, also called heat, is related to temperature in that heat always flow from higher temperature to lower temperature. When two objects are at the same temperature, they are said to be at thermal equilibrium.

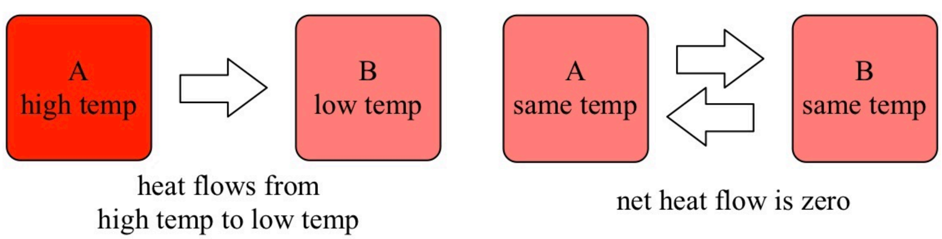

FIG. 6.1 DIRECTION OF HEAT FLOW

Temperature determines the direction of heat flow. Heat always flow from a higher temperature to a lower temperature.

When thermal equilibrium is reached, the rate of heat flow from A to B is the same as the rate of heat flow from B to A. Hence, there is no net increase in heat energy in either bodies.

6.2 Celsius and the kelvin scale

The Celsius scale is based on 100 equal divisions calibrated between melting point of ice and boiling point of water. Since this scale is calibrated using a real thermometer using a specific thermometric property, the temperature read by the thermometer will not be consistent with the value obtained by another type of thermometer.

Thermometric property

A **thermometric property** is a property of a material that changes with to temperature.
To eliminate this problem of inconsistency, the thermodynamic scale is created. The thermodynamic scale is not based on thermal property and it uses the absolute zero and the triple point of water as the lower and upper fixed points.

$$T/\mathrm{K} = T/^\circ\mathrm{C} + 273.15$$

Absolute zero is the temperature which any substance has the lowest possible energy state. Triple point of water is the temperature which the three states of water exist in equilibrium.
A difference of 1 K is identical to a difference of 1 ºC.

$$1\ \mathrm{K} = 1^\circ\mathrm{C}$$

Thermocouple

A thermocouple uses the voltage generated between two metal junctions to measure temperature. When the thermocouple is calibrated, it can be used to measure temperatures up to hundreds of degree celsius.

Resistance thermometer and thermistor

Resistance thermometer uses the resistance property of a metallic wire to measure temperature. A thermistor is a semiconductor where its resistance also changes with temperature.

Thermocouple	Resistance thermometer	Thermistor
voltage produced between hot and cold junctions	changes in resistance of a metallic wire	changes in resistance of a semiconductor
larger temperature difference between hot and cold junction produces larger voltages	higher temperature produces higher resistance	higher temperature produces lower resistance
high sensitivity	high sensitivity	high sensitivity

Table 6.1 Comparison of three types of thermometers

7

Thermal properties of matter

7.1 Three states of matter

Matter exists in three states. Only in the gaseous state is the intermolecular/interatomic forces negligible. In both solid and liquid, there exists significant intermolecular/interatomic forces.

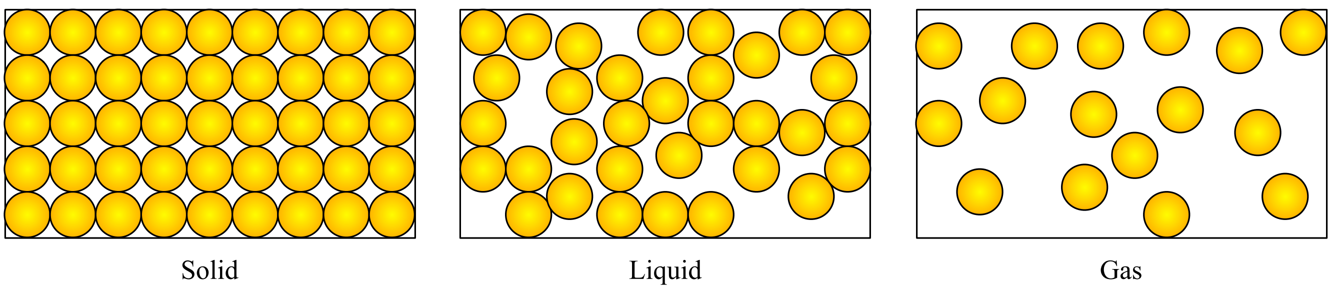

FIG. 7.1 STRUCTURE OF THE THREE STATES OF MATTER

	distance between particles	structure of particle arrangement	motion of particles
Solid	Close	Regular	Vibration
Liquid	Close	Irregular	Random sliding over one another
Gas	Far, widely separated	Irregular	Random and fast moving, changing direction only upon collisions

TABLE 7.1 COMPARISON OF THE STRUCTURE AND MOTION OF PARTICLES IN THE THREE STATES OF MATTER.

7.2 Specific heat capacity

Heat energy supplied in the phases between melting and boiling is used to increase the temperature of the substance. The average speed of the particles increases due to the increase in internal energy.

> ***Specific heat capacity*** *of a substance is the energy needed to increase the temperature of a unit mass of substance by 1 kelvin.*

$$Q = mc\Delta\theta \qquad (7.1)$$

where Q is the heat energy, m is the mass of the substance, c is the specific heat capacity and $\Delta\theta$ is the change in temperature of the substance.

The SI unit of c is J kg^{-1} K^{-1}.

When the entire substance is concerned, the **heat capacity** C is used.

$$C = mc \qquad (7.2)$$

Experiment to determine specific heat capacity

The method to produce heat is always by electrical means because the quantity of heat supplied must be known accurately. To measure the specific heat capacity of a substance, the following may be done:

1. A heater coil is connected to a power supply to generate heat at a power of P for a time t. The heat generated is Pt.
2. The mass of the object, m is measured.
3. The temperature of the object before the heater is turned on T_i and the temperature after time t, T_f is measured. The difference of these two temperatures are calculated $\Delta\theta = T_i - T_f$.
4. Specific heat capacity is calculated using $c = \dfrac{Pt}{m\Delta\theta}$.

Lagging should be done to make the result more accurate. Heat lost to the surrounding causes the measured c to be higher than the true value.

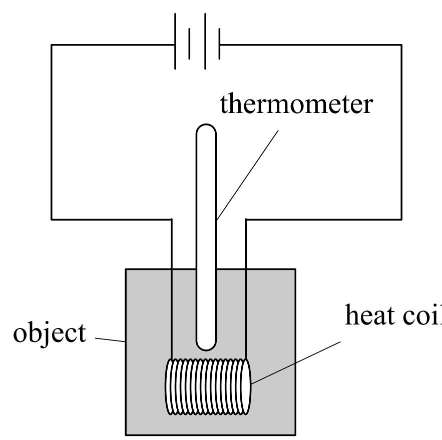

FIG. 7.2 EXPERIMENTAL SETUP TO MEASURE THE SPECIFIC HEAT CAPACITY OF A SUBSTANCE.

7.3 Specific latent heat

Melting

A solid absorbs enough heat energy to melt into liquid. Heat energy is used to break the forces between molecules of a solid. The average distance between atoms is slightly further apart in liquid than in solid. There are still significant intermolecular forces between molecules in liquid. The average speed of the molecules during melting does not increase with the supplied heat, hence temperature does not increase.

> ***Specific latent heat of fusion*** *is the quantity of heat energy required to convert unit mass of solid to liquid without any change in temperature*

$$Q = m l_f \tag{7.3}$$

where Q is heat energy, m is the mass of the substance and l_f is the specific latent heat of fusion of the substance.

Boiling

Heat energy is used to completely break the forces between the atoms. The atoms are not bonded to each other in a gas. The average distance between atoms is large.

Besides breaking the intermolecular forces, heat energy is also used to do work against external forces (atmospheric pressure) during the expansion of liquid into gas. As such, the specific latent of vaporisation is much larger than the specific latent heat of fusion.

The average speed of the molecules during boiling/vaporisation does not increase with the supplied heat, hence temperature does not increase.

> ***Specific latent heat of vaporisation*** *is the quantity of heat energy required to convert unit mass of liquid to gas without any change in temperature.*

$$Q = m l_v \tag{7.4}$$

where Q is heat energy, m is the mass of the substance and l_v is the specific latent heat of vaporisation of the substance.

Evaporation

During evaporation, the higher speed particles at the surface of the liquid escape from the liquid to the air. As a result, the average speed of the particles left is smaller, causing a lowering of temperature in the liquid. Thus, evaporation causes cooling.

Evaporation is NOT boiling and there are differences between the two processes.

boiling	evaporation
requires heat to be supplied	does not requires heat supply
temperature remains constant	causes cooling
bubbles produced	no bubbles produced

TABLE. 7.2 DIFFERENCES BETWEEN BOILING AND EVAPORATION.

Overview of the process of heat supply to melt and boil a substance

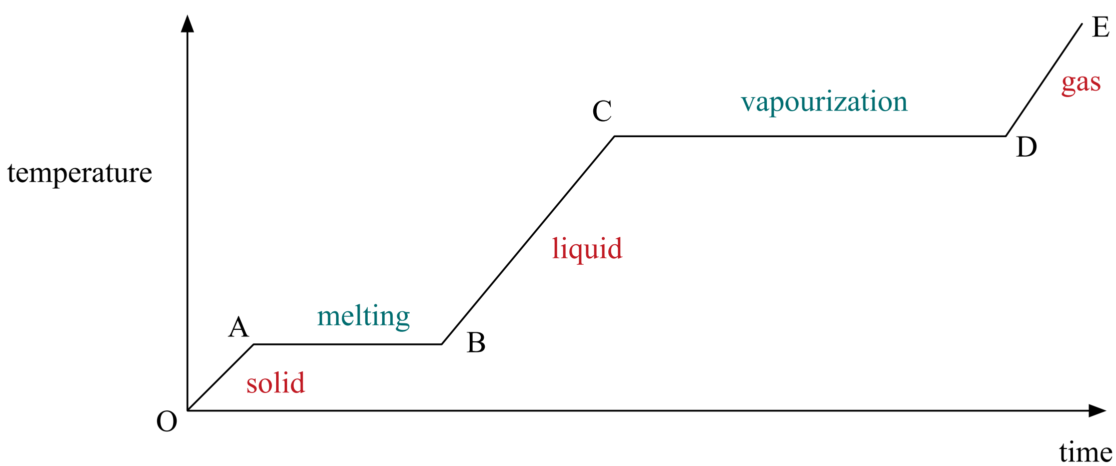

FIG. 7.3 TEMPERATURE RISE WHEN A SUBSTANCE IS HEATED CONTINUOUSLY

Stage	Event	State/s present	Note
OA	Temperature rising	Solid	
AB	Melting. Temperature constant	Mixture of solid and liquid	Energy to break solid bonds
BC	Temperature rising	Liquid	
CD	Vaporisation. Temperature constant	Mixture of liquid and gas	Requires more heat than AB
CE	Temperature rising	Gas	

TABLE 7.3 DESCRIPTION OF SUBSTANCE AT DIFFERENT PARTS DURING THE HEATING PROCESS.

Experiment to determine the specific latent heat

Similar to the experiment to measure specific heat capacity, the heat source should be electrical in nature so that the quantity can be measured accurately.

1. The power source is turned on. When the timer is started, the mass of the liquid m_i is measured.
2. After time t, the mass of the liquid remaining m_f is measured. The mass of liquid changes to gas is $m_f - m_i$.
3. The heat quantity supplied to boiled away the liquid is Pt, where P is the power of the heat supply.
4. Specific latent heat of vaporisation is calculated using $l_v = \dfrac{Pt}{m_f - m_i}$.

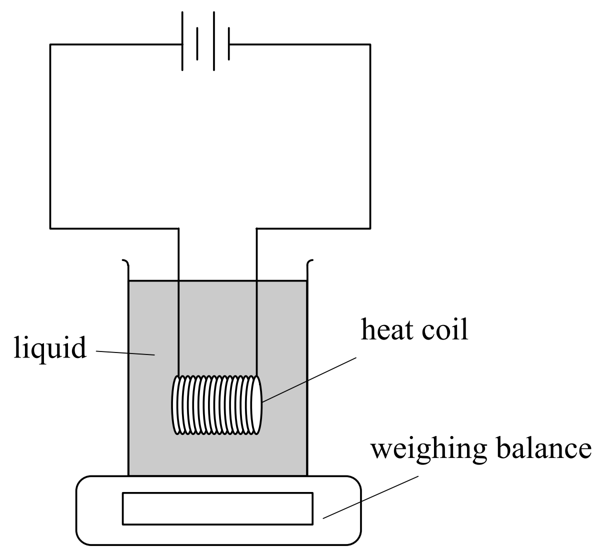

FIG. 7.4 EXPERIMENTAL SETUP TO MEASURE THE SPECIFIC LATENT HEAT OF VAPORISATION OF A LIQUID.

Again, heat lost to surrounding need to be accounted for. This may be done using one of the two methods:

Lagging

Lagging reduces heat lost to surrounding. If this is not done, heat lost will cause the measured l_v to be higher than the true value.

Repeating

By repeating the experiment, heat lost can be eliminated.

$$
\begin{aligned}
Pt_1 &= \Delta m_1 l_v + Q_1 \\
Pt_2 &= \Delta m_2 l_v + Q_1 \\
P(t_1 - t_2) &= (\Delta m_1 - \Delta m_2) l_v \\
\therefore l_v &= \frac{P(t_1 - t_2)}{\Delta m_1 - \Delta m_2}
\end{aligned}
$$

This method of heat loss elimination is possible here because the temperature of the substance remains constant. This means that the rate of heat loss to the surrounding is constant. This method cannot be used to determine the specific heat capacity because the temperature of the substance is not constant and the rate of heat loss is not constant.

7.4 First law of thermodynamics

The molecules in a system are always in constant motion. Each molecule has kinetic energy due to this motion. In addition, the intermolecular forces result in the presence of potential energies of these individual molecules. The sum of these kinetic energies and potential energies of the molecules contribute to the **internal energy** of the molecules.

The sum of kinetic energies and potential energies of the random ensemble of molecules in a system is the internal energy of the system.

The internal energy of the system increases when the average kinetic energy of the molecules in the system increases. As a result, an increase of internal energy will cause the temperature of the system to increase as well.

$$\Delta U = Q + W$$

where ΔU is the **CHANGE** in internal energy of the system, Q is the heat supplied **TO** the system and W is the work done **ON** the system. In other words, what this equation is saying is that to increase the internal energy of the system, there are only two ways to do it:

1. supply heat to the system, or
2. do work on the system.

P-V diagram

A pressure-volume graph is usually used to represent the changes occurring to an ideal gas system.

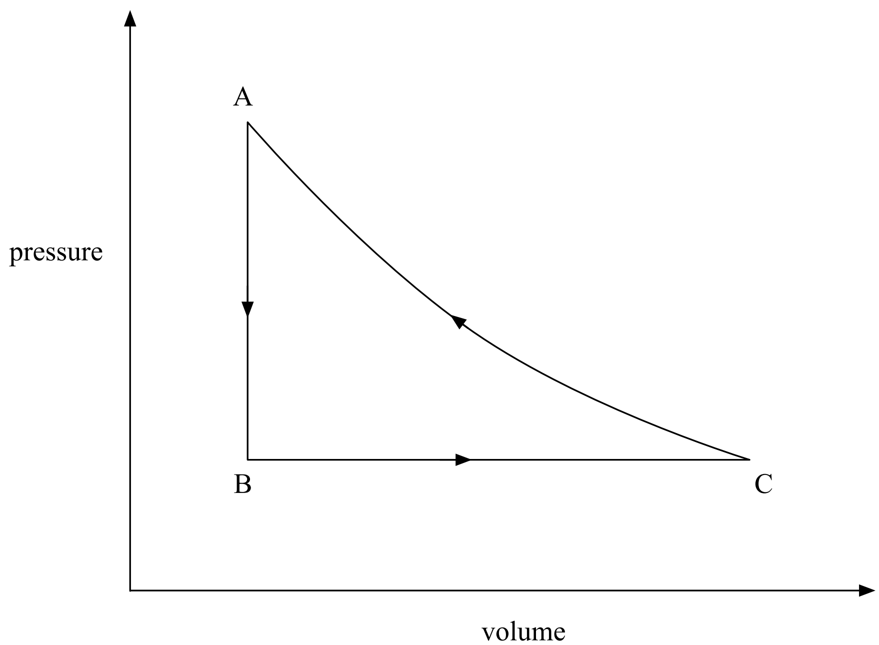

FIG. 7.5 A TYPICAL EXAMPLE OF A P-V GRAPH.

1. Each point on a *P-V* graph represents a particular temperature. This is because $PV = nRT$. Each unique combination of P and V has a unique T.
2. AB is a constant-volume process. Since pressure decreases, we can conclude that heat energy is lost during the process, thus internal energy decreases.
3. BC is a constant-pressure process. Since the volume increases, we can conclude that heat energy is supplied such that the volume expands while keeping the pressure constant. If heat is not supplied, volume expansion will cause temperature to decrease because work is done by the system on the external environment.
4. CA is a process with volume contraction while pressure increases.

5. Since a particular T represents a particular internal energy of the system, a closed series of processes such as ABCA means that there is no change in internal energy of the system. The initial internal energy of the system at A is the same as the final internal energy at A after going through AB, BC and CA.
6. A point that is higher or on the right has a higher internal energy than a point below or to the left. Thus, A has a higher internal energy than B while C has a higher internal energy than B. You can't conclude whether A or C has higher internal energy by just looking at the points.
7. The area under the graph represents the work done on/by the system. The area enclosed by the entire process ABCA is the net work done on the gas.

About the author

Stanley Sim teaches Physics and Mathematics at middle and high school levels for 10 years. He is currently teaching Physics and is the director at Classroom Technologies LLP.

Stanley is an advocate of using technology for learning. He is an Apple Distinguished Educator and uses Mac since the days of Mac OS 9.

During his free time, Stanley loves to code in Ruby on Rails and Swift. He has also experiences in teaching C, C++ and PHP. He created a result management system with FileMaker Pro that his school used to manage results across multiple campuses.

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