STANLEY SIM

$$mg - R = \frac{\Delta p}{\Delta t}$$

AS PHYSICS

General Physics

Stanley's Study Guides

Classroom Technologies

AS Physics

General Physics

written and illustrated by

Stanley Sim

Classroom
Technologies

Contents

Preface

A-level Physics is a hard subject. It is hard because there is a lot of contents and the problems are hard to solve. In this series of Stanley's Study Guides, I attempt to summarise the concepts found in the Cambridge International Examination's A-level Physics syllabus (code 9702). This is to allow students to focus on the key concepts and prepare well for their examinations.

This volume on *General Physics* covers the following topics of the A2-level Physics curriculum in the Cambridge International Examinations:

- Circular motion
- Gravitational fields
- Electric fields
- Simple harmonic motion
- Ideal gas
- Temperature
- Thermal properties of matter

All the best to your study.

Thank you
Stanley
June 2018

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Unit and uncertainty

1.1 Unit prefixes

Prefixes are used to simplify the representations of very large or small numbers. They reduce the number of zeros in a quantity.

prefix	symbol	value	example
pico	p	$\times 10^{-12}$	picometer(pm)
nano	n	$\times 10^{-9}$	nanometer(nm)
micro	µ	$\times 10^{-6}$	micrometer(µm)
milli	m	$\times 10^{-3}$	millimeter(mm)
centi	c	$\times 10^{-2}$	centimeter(cm)
deci	d	$\times 10^{-1}$	decimeter(dm)
kilo	k	$\times 10^{3}$	kilometer(km)
mega	M	$\times 10^{6}$	megameter(Mm)
giga	G	$\times 10^{9}$	gigameter(Gm)
tera	T	$\times 10^{12}$	terameter(Tm)

TABLE 1.1 THE TEN PREFIXES ARE USED TO SHORTENED NUMERICAL REPRESENTATIONS.

1.2 Base units

The 6 base units in **Table 1.2** are used extensively throughout the study of this course. These base units are often combined to form derived units.

Physical quantity	SI unit	symbol
mass	kilogram	kg
length	metre	m
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol

TABLE 1.2 SIX BASE PHYSICAL QUANTITIES

Take particular note that only mass has a prefix of kilo in its SI unit. The SI units of other physical quantities has no prefixes.

Also note that a 7th physical quantity luminosity, is not in the syllabus.

1.3 Derived Units

Derived units are units based on the combinations of two or more base units. Some examples are speed and volume.

Follow the procedure to find the unit of a physical quantity:

1. Express the relevant physical quantity as the subject of the formula.
2. Replace the physical quantities by their respective base units. If a quantity is a derived quantity and is not trivial to express its unit as base units, then you may need to perform additional steps(#1, #2 and #3) until you found its unit in terms of base units.
3. Evaluate.

EXAMPLE 1

Find the base units of speed.

Solution

Step 1: Write down the formula of speed.

$$v = \frac{s}{t}$$

Step 2: Replace the physical quantities with the units.

$$\text{unit of speed} = \frac{\text{unit of distance}}{\text{unit of time}}$$

Step 3: Evaluate.

$$\text{unit of speed} = \frac{\text{m}}{\text{s}} = \text{m s}^{-1}$$

EXAMPLE 2

Find the base units of force.

Solution

Step 1: Write down the formula of force.

$$F = ma$$

Step 2: Replace the physical quantities with the units

$$\text{unit of force} = \text{unit of mass} \times \text{unit of acceleration}$$

Step 3: Evaluate.

$$\begin{aligned}\text{unit of force} &= \text{kg} \times \text{m s}^{-2} \\ &= \text{kg m s}^{-2}\end{aligned}$$

EXAMPLE 3

Young modulus is the ratio of stress to strain. Stress is defined as the ratio of force per unit cross-sectional area while strain is the ratio of extension to the original length.

Find the base units of Young modulus.

Solution

Step 1: Write down the formula for Young modulus.

$$E = \frac{\text{stress}}{\text{strain}}$$

Step 2: Replace the physical quantities with the units.

We can work out the units of stress and strain separately because they are not trivial.

$$\begin{aligned}\text{stress} &= \frac{\text{force}}{\text{area}} \\ &= \frac{F}{A}\end{aligned}$$

$$\begin{aligned}\text{units of stress} &= \frac{\text{kg m s}^{-2}}{\text{m}^2} \\ &= \text{kg m}^{-1}\text{s}^{-2}\end{aligned}$$

$$\text{strain} = \frac{\text{extension}}{\text{original length}}$$

$$\begin{aligned}\text{unit of strain} &= \frac{\text{m}}{\text{m}} \\ &= \text{no unit}\end{aligned}$$

$$\therefore \text{unit of E} = \text{kg m}^{-1}\text{s}^{-2}$$

1.4 Unit consistency

Whenever two quantities of the same nature exists in an equation, care must be taken to ensure that they have the same prefix.

EXAMPLE 4

Calculate the average speed of the car given that it travelled the journey in two parts:

Part 1: 2.5 km in 2.0 min

Part 2: 1500 m in 1.5 min

Solution

$$\begin{aligned}\text{Total distance travelled} &= 2500 + 1500 \\ &= 4000 \text{ m} \\ \text{Total time taken} &= 120 + 90 \\ &= 210 \text{ s} \\ \text{average speed} &= \frac{\text{total distance travelled}}{\text{total time taken}} \\ &= \frac{4000}{210} \\ &= 19.0 \text{ m s}^{-1}\end{aligned}$$

1.5 Homogeneity of Equation

In any equation where each term has the same base units, the equation is said to be homogeneous.

If $A + B = CD$, then

$[A] = [B] = [CD]$

i.e. the unit of A = the unit of B = the unit of CD

The square brackets represent "the units of".

Every equation must be homogeneous.

- An equation that is homogeneous may not be correct.
- If an equation is not homogeneous, then the equation must be invalid.

To illustrate the above statements,

- $E_K = \frac{1}{2}mv^2$: this equation is homogeneous and is true.
- $E_K = mv^2$: this equation is homogeneous but is false.
- $E_K = \frac{1}{2}mv^3$: this equation is not homogeneous and is false.

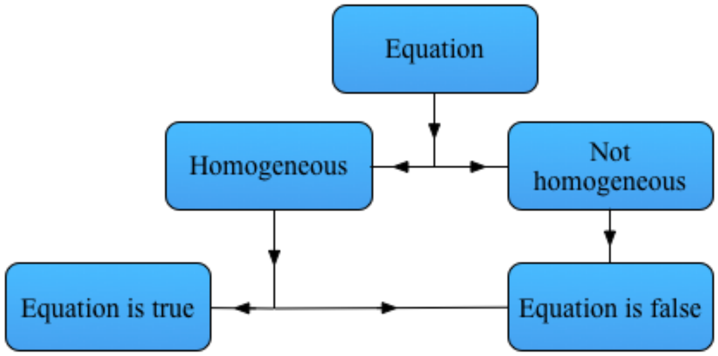


FIG. 1.1 HOW AN EQUATION IS TRUE OR NOT RELATE TO THE HOMOGENEITY OF THE EQUATION

A homogeneous equation may be true or false, but a non-homogeneous equation but be false.

EXAMPLE 5

Froude number is the ratio of the inertia forces to the gravitational forces and it has a formula

$$F_r = \frac{v^n}{\sqrt{gl}}$$

where v is the velocity of the object, g is the gravitational acceleration and l represents length. n is a constant with no unit.

Determine the value of n.

Solution

The strategy is to equate the unit of the LHS of equation to the RHS of equation since in a valid equation, the units of LHS must be consistent with the units of RHS.

Froude number has no unit because it is a ratio of force to force. Note that being a ratio is NOT the reason for it to have no unit. A ratio can have units e.g. speed is the ratio of distance to time.

Now, we have to work on the unit of RHS

$$\text{unit of RHS} = \frac{\text{m}^n\ \text{s}^{-n}}{\sqrt{\text{m s}^{-2} \times \text{m}}}$$

$$= \frac{\text{m}^n\ \text{s}^{-n}}{\sqrt{\text{m}^2\ \text{s}^{-2}}}$$

$$= \frac{\text{m}^n\ \text{s}^{-n}}{\text{m s}^{-1}}$$

$$\therefore n = 1$$

Quantity	Instrument	Precision
Length	metre rule	1 mm
	vernier callipers	0.1 mm
	micrometer screw gauge	0.01 mm
Pressure	manometer	1 mm
	barometer	1 mm
Mass	top-pan electronic balance	0.01 g
	spring balance	varies e.g. 1 g
Angle	protractor	1°, 0.5°
Time	stopwatch	0.01 s
Temperature	mercury-in-glass thermometer	1°C, 0.5°C
	thermocouple	0.1 °C
Current	analogue ammeter	0.01 A to 0.1 A
	digital ammeter	0.01 A
Volt	analogue voltmeter	0.1 V - 0.2 V
	digital voltmeter	0.01 V

TABLE 1.3 PRECISION OF SOME COMMON MEASURING INSTRUMENTS

1.6 Measurements

Table 1.3 shows the precision of some common measuring instruments. The precision of a measuring instrument determines how you will record the values. From reading the recorded values, you will be able to determine the precision of the instrument used. Hence it is important that when recording a measured reading, care must be taken to include the necessary decimal digits.

1.7 Systematic and Random Errors

Systematic error

A systematic error results in all readings being above or below the true value. This error cannot be eliminated by repeated readings and taking average. Systematic error can only be reduced by improving experimental techniques or by accounting for the error (such as adding or subtracting the error).

Examples of systematic errors

- Zero error
- Wrongly calibrated scale
- Un-tared reading of mass on an electronic balance

Random Error

Random error results in readings being scattered around a mean value. Random error may be reduced by repeating a reading and averaging, or by plotting a graph and drawing a best-fit line.

Examples of random errors

- timing oscillations of pendulum
- taking readings of a quantity that varies with time
- reading a scale from different angles (parallax error)

Accuracy and precision

Accuracy refers to how close the experimental values are to the true value. If the average of the data is close to the true value, then the data is said to be accurate.

Precision refers to how close the experimental values relate to each other. If the difference between the values are small, the data is said to be precise.

1.8 Uncertainty

Uncertainty of a measured quantity depends on a few factors:

1. If the instrument has a fixed end, uncertainty takes the value of half the smallest interval of the measuring instrument. Instruments with fixed ends are
   (i) measuring cylinders
   (ii) liquid-in-glass thermometers
2. If both ends of the instruments are needed to measure the item, the uncertainty takes the value of the smallest interval. Ruler belongs to this category i.e. uncertainty = 1 mm.
3. Digital meters has uncertainty that equals to the least significant digit.

instrument	uncertainty	typical reading	wrong reading
liquid-in-glass thermometer	±0.5 °C	22.5 °C, 24.0 °C	22.3°C
analogue ammeter with 0.1 A division	±0.05 A	0.15 A, 1.20 A	0.17 A
ruler with 1 mm intervals	±0.1 cm	5.7 cm, 10.0 cm	10.00 cm
digital stopwatch	±0.01 s	16.22 s, 15.01 s, 17.79 s	16.220 s

TABLE 1.4 THIS TABLE SHOWS HOW SOME MEASUREMENTS ARE WRITTEN ACCORDING TO THE MEASURING INSTRUMENTS.

1.9 Uncertainties of a derived quantity

Two rules to obtain the uncertainty of a derived quantity:

Rule 1

For quantities which are added or subtracted to give a final result, add the absolute uncertainties.

$$\begin{aligned} x &= y \pm 2z \\ \Delta x &= \Delta y \pm 2\Delta z \end{aligned} \tag{1.1}$$

Notes:

1. Absolute uncertainties are always added up, even if the quantities are subtracted e.g. $y - 2z$.
2. Multiplying factors in the physical quantities ($2z$) are also multiplied in the absolute uncertainties ($2\,\Delta z$).

Rule 2

For quantities which are multiplied or divided to give a final result, add the fractional uncertainties. Fractional uncertainty is the fraction of the actual uncertainty divide by the data value.

$$\begin{aligned} A &= \frac{3B^2}{C} \\ \frac{\Delta A}{A} &= 2\frac{\Delta B}{B} + \frac{\Delta C}{C} \end{aligned} \tag{1.2}$$

Notes:

1. Fractional uncertainties are used in formula that involves multiplication or division.
2. Since $B^n = B \times B \ldots \times B$, the power is bought down as the multiplying factor in the fractional uncertainty.
3. Multiplying factors are ignored in the fractional uncertainty.

1.10 Expressing a quantity with its uncertainty

Uncertainty is usually expressed to one significant figure and the measured quantity is expressed to the same precision as the uncertainty.

i.e. if the uncertainty is calculated to be 0.17 cm s^{-1} and the measured quantity is 23.4 cm, the the correct way to express the quantity is

$$(23.4 \pm 0.2) \text{ cm s}^{-1}$$

Notes:

1. The uncertainty 0.17 cm s^{-1} is written to 1 significant figure i.e. 0.2 cm s^{-1}.
2. The uncertainty 0.2 cm s^{-1} has a 0.1 cm s^{-1} precision(one decimal place). Hence, the main quantity is written to the same precision i.e. 23.4 cm s^{-1}(one decimal place).

End of chapter

2

Kinematics

2.1 Equations of motion

Definition of average speed:

Average speed is defined as the total distance covered in the time taken.

$$\text{average speed} = \frac{\text{total distance}}{\text{time taken}} \tag{2.1}$$

The SI unit of speed is metres per second .

EXAMPLE 1

The radius of the Earth is 6400 km. One revolution about its axis takes 24 hours. Calculate the average speed of a point on the equator relative to the centre of the Earth.

Solution

$$\begin{aligned}
\text{average speed} &= \frac{\text{total distance}}{\text{total time}} \\
&= \frac{2\pi r}{t} \\
&= \frac{2\pi(6400)}{24 \times 3600} \\
&= 0.461 \text{ km s}^{-1}
\end{aligned}$$

2.2 Speed and velocity

Speed is a scalar quantity. It has magnitude only.

Speed is distance travelled per unit time as shown in **Eq (2.1)**.

Velocity is a vector quantity. It has both magnitude and direction.

Velocity is displacement travelled per unit time.

$$\text{average velocity} = \frac{\text{displacement}}{\text{time taken}} \qquad (2.2)$$

The important point regarding this formula is that it can only be used to calculate the average velocity of a particular journey and not the initial or the final velocity in a linear motion with acceleration.

2.3 Acceleration

Definition of acceleration:

Acceleration is the rate of change of velocity.

$$a = \frac{v_2 - v_1}{t_2 - t_1} \qquad (2.3)$$

An object is said to be accelerating when its velocity changes:

- only the speed changes e.g. the object is moving faster or slower.
- only the direction of motion changes e.g. circular motion.
- both speed and direction changes e.g. projectile motion.

Example 2

A sprinter, starting from the blocks, reaches his full speed of 9.0 m s^{-1} in 1.5 s.

What is the average acceleration?

Solution

$$\begin{aligned} a &= \frac{\text{change in velocity}}{\text{time taken}} \\ &= \frac{v - u}{t_2 - t_1} \\ &= \frac{9.0 - 0}{1.5} \\ &= 6.0 \text{ m s}^{-2} \end{aligned}$$

EXAMPLE 3

A car is travelling at a constant speed of 25 m s^{-1}. It then increases its speed to 30 m s^{-1} in 2.5 s. What is the average acceleration of the car?

Solution

$$\begin{aligned} a &= \frac{v-u}{t_2-t_1} \\ &= \frac{30-25}{2.5} \\ &= 2.0 \text{ m s}^{-2} \end{aligned}$$

2.4 Deriving the equations of kinematics

From the **Eq (2.3)**,

$$a = \frac{v-u}{t}$$

$$v = u + at \tag{2.4}$$

where v is the final velocity, u is the initial velocity and t is the time taken for the acceleration.

For an object moving with constant acceleration a, the average velocity can be expressed using the relationship

$$v_{avg} = \frac{s}{t} = \frac{u+v}{2}$$

$$\begin{aligned} 2s &= ut + vt \\ &= ut + (u+at)t \\ &= 2ut + at^2 \end{aligned}$$

$$s = ut + \tfrac{1}{2}at^2 \tag{2.5}$$

From **Eq (2.4)**, replace t by $\frac{v-u}{a}$,

$$s = u\left(\frac{v-u}{a}\right) + \tfrac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

$$v^2 = u^2 + 2as \tag{2.6}$$

These three equations of kinematics are only applicable if the object is moving at constant acceleration.

To summarise,

$$v = u + at \tag{2.4}$$

$$s = ut + \frac{1}{2}at^2 \tag{2.5}$$

$$v^2 = u^2 + 2as \tag{2.6}$$

In all these equations, the symbols represent:

v: final velocity

u: initial velocity

s: displacement

a: acceleration

t: time taken

Example 4

A ball is dropped from rest at a height of 2.0 m. Calculate the time it takes for the ball to reach the ground.

Solution

$u = 0\ \text{m s}^{-1}$

v: not applicable

$s = 2.0\ \text{m}$

$a = 9.81\ \text{m s}^{-2}$

t: to calculate

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ 2.0 &= \frac{1}{2}(9.81)t^2 \\ t &= 0.64\ \text{s} \end{aligned}$$

EXAMPLE 5

A plane must reach a speed of 100 m s^{-1} to take off. If the available length of the runway is 2.4 km and the aircraft accelerated uniformly from rest at one end, what minimum acceleration must be attained if it is to take off?

Solution

$u = 0$ m s^{-1}

$v = 100$ m s^{-1}

$s = 2400$ m

a: to calculate

t: not applicable

$$v^2 = u^2 + 2as$$
$$a = \frac{v^2 - u^2}{2s}$$
$$= \frac{100^2 - 0^2}{2 \times 2400}$$
$$= 2.1 \text{ m s}^{-2}$$

EXAMPLE 6

A ball is thrown vertically upwards with a speed of 5.0 m s^{-1}. What is its velocity when it first passes through a point 1.0 m above the cricketer's hands?

Solution

This is the first scenario where the acceleration is in opposite direction to the initial velocity of the object. In such situations, a negative needs to be appended to the acceleration value i.e. the object is decelerating.

$u = +5$ m s^{-1}

v: to calculate

$s = 1.0$ m

$a = -9.81$ m s^{-2}

t: not applicable

$$v^2 = u^2 - 2as$$
$$= 5.0^2 - 2(9.81)(1.0)$$
$$v = 2.3 \text{ m s}^{-1}$$

2.5 Graphs of motions

There are in general, two types of graphs related to motion:

- displacement-time
- velocity-time

displacement-time graph

- The gradient at a point represents the instantaneous velocity at that point.
- In **Fig. 2.1**, the velocity at P can be found from the gradient of the tangent at P.

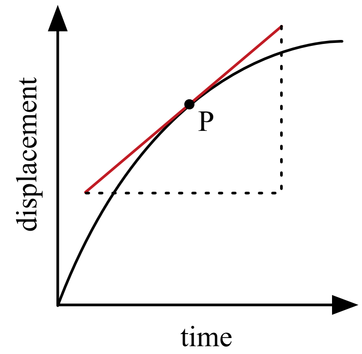


FIG. 2.1 DISPLACEMENT-TIME GRAPH

The gradient of an *s*-*t* graph represents the velocity of the body.

velocity-time graph

- Gradient at a point represents acceleration at the point.
- Area under graph represents the displacement travelled.

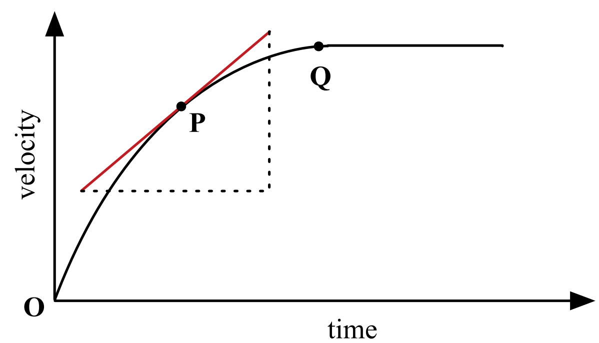


FIG. 2.2 VELOCITY-TIME GRAPH

The gradient of a *v*-*t* graph represents the acceleration of the body.

The acceleration of the body at point P in **Fig. 2.2** can be obtained by find out the gradient of the tangent at point P. Since the gradient decreases from O to Q, the acceleration of the body also decreases from O to Q. After point Q, the body moves with constant velocity i.e. zero acceleration.

Fig. 2.3 shows a very common velocity-time graph is that of a body falling freely from rest. Since the gradient represents the acceleration due to gravity, the gradient of OA is 9.81 m s^{-2}. The direction of the acceleration is down, so the gradient should be negative. The distance the body fell during this time is represented by the shared area.

Another scenario is an object moving upwards with a velocity, reaches its maximum height, and fall back to its starting position, in the absence of air resistance. Similar to the first scenario, the gradient represents the acceleration of gravity and it should be a linear straight line with a negative gradient if value 9.81 m s^{-2}.

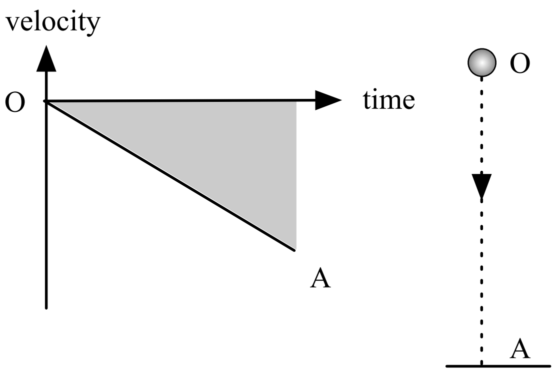


FIG. 2.3A V-T GRAPH OF A BODY FALLING FROM REST

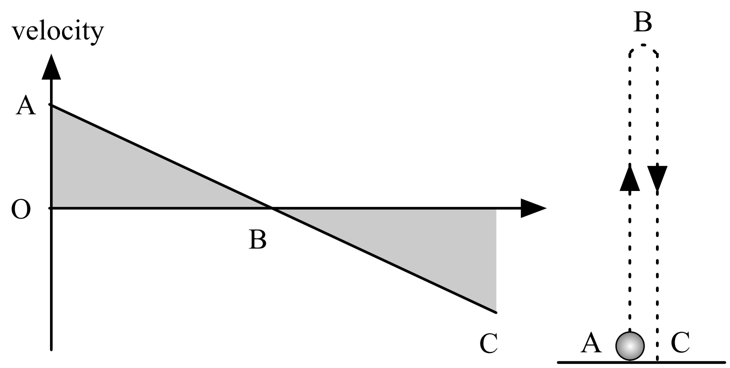


FIG. 2.3B V-T GRAPH OF A BODY THROWN UPWARDS AND FALLING FREELY UNDER GRAVITY.

2.6 2D motion – projectile motion

When an object travels with a uniform velocity in one direction, a perpendicular force causes its direction to deviate from the original direction.

To solve problems involving this type of motion, resolve the velocity into two directions:

Parallel to the direction of force

The motion is equivalent to a uniform acceleration in the direction of the force. The three kinematics formulae should be used:

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

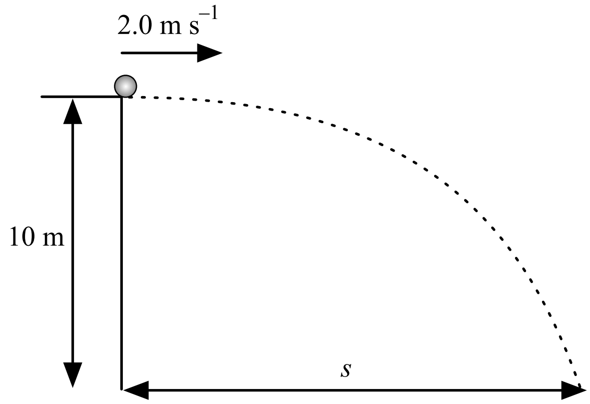
Perpendicular to the direction of force

Velocity remains constant in this direction.

Use " $v = \frac{s}{t}$ ".

EXAMPLE 7

A ball was thrown horizontally from a 10 m height.



Calculate

(i) the time it took to hit the ground,

Vertically:

$u = 0 \text{ m s}^{-1}$

v: not applicable

$s = 10 \text{ m}$

$a = 9.8 \text{ m s}^{-2}$

t: to calculate

$s = ut + \frac{1}{2}at^2$

$$t = \sqrt{\frac{2(s - ut)}{a}}$$

$$= \sqrt{\frac{2(10 - 0)}{9.81}}$$

$$= 1.4 \text{ s}$$

(ii) the horizontal distance s.

Solution

Since it took 1.4 s for the ball to hit the ground, it also took the ball the same time to travel s horizontally since it is the same ball.

Horizontally, there is no acceleration. Hence, we use $s = vt$.

$$s = vt$$
$$= 2.8 \text{ m}$$

2.7 Resolving velocity into two directions

Velocity could be resolved into two perpendicular directions when solving a 2D projectile motion problem. The velocity vector must always be the hypotenuse of the right-angle triangle.

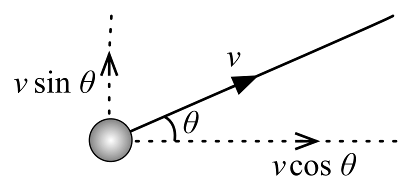
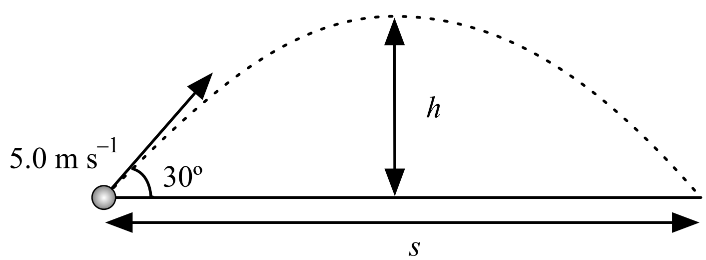


FIG. 2.4 RESOLVING A VECTOR INTO TWO PERPENDICULAR DIRECTIONS.

EXAMPLE 8

A ball was thrown at an angle of 30º from the horizontal with a speed of 5.0 m s^{-1}.



Calculate

(i) the maximum height that the ball reached,

Solution

Vertically,

$u = 5.0 \sin 30^\circ = 2.5 \text{ m s}^{-1}$

$v = 0 \text{ m s}^{-1}$

s: to calculate

$a = 9.81 \text{ m s}^{-2}$

t: not applicable

vertical component of velocity $= 5.0 \sin 30^\circ = 2.5 \text{ m s}^{-1}$

$$v^2 = u^2 + 2as$$
$$0^2 = 2.5^2 - 2(9.81)s$$
$$s = 0.32 \text{ m}$$

(ii) the horizontal distance when the ball touched the ground.

Solution

Horizontally,

$$v = u + at$$
$$0 = 2.5 - 9.81t$$
$$t = 0.25 \text{ s}$$

horizontal component of velocity $= 5.0 \cos 30^\circ = 4.3 \text{ m s}^{-1}$

$$s = 2 \times 4.3(0.25) = 2.2 \text{ m}$$

End of Chapter

3

Dynamics

3.1 The concept of mass

Mass is the property of a body that resists change in motion.

A body tends to continue its motion in the same way. If a body is at rest, it will tend to continue at rest. If it is moving, it will tend to continue its motion at the same speed in the same direction.

3.2 Newton's first law

Definition of Newton's first law:

> *An object will continue its state of motion at the same velocity in the absence of a resultant force acting on it.*

If there is no net force acting on a body, it will continue its current state of motion. That means it will continue to move at a constant speed in the same direction. It should not slow down unless there is an opposing force.

3.3 Linear momentum

Definition of momentum:

> *Momentum is defined as the product of mass of the body and its velocity.*

$$p = mv \tag{3.1}$$

where p is the momentum, m is the mass and v is the velocity. The SI unit of momentum is kg m s^{-1}.

3.4 Newton's second law

The resultant force acting on a body is directly proportional to the acceleration of the body. The constant of proportionality is mass of the body.

$$F = ma \tag{3.2}$$

where F is the resultant force, a is the acceleration and m is the mass of body. The SI unit of force is newton (N).

Note that F and a are always in the same direction.

Force and velocity

The most common misconception by students is that the resultant force is in the same direction as the velocity. This idea is completely incorrect. Velocity has no direct relationship with force.

In **Fig. 3.1**, the ball is thrown upwards at point A with an initial velocity of $+v$. However, there is only one force, that is the weight w, acting on the ball down. Similarly, the velocity at point C is $-v$, down and the weight is also downwards. Velocity is 0 at point B while the weight is still acting downwards. This shows that force and velocity has no direct relationship with each other.

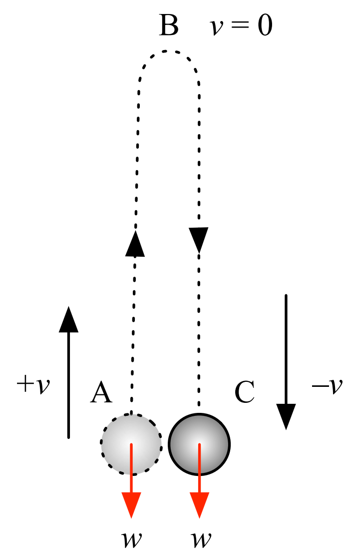


Fig. 3.1 Relationship between force and velocity at different parts of a motion

This diagram shows that velocity and the net force acting on an object can have different directions. Net force in the same direction of the velocity cause it to accelerate while net force in opposite direction of the velocity causes it to decelerate.

point	net force (w)	velocity (v)
A	down	up
B	down	0
C	down	down

Table 3.1 This table summarises the directions of the net force and the velocity of a ball thrown upwards and falls freely under gravity.

Definition of Newton’s second law:

The resultant force acting on an object is directly proportional to the rate of change of momentum.

$$F = \frac{\Delta p}{\Delta t} \tag{3.3}$$

where F is the resultant force acting on an object, Δp is the change in momentum and Δt is the time taken for the momentum change[1].

It is important to note that when p and t are expressed in SI units, the constant of proportionality would be 1. Hence the "equal" sign for the Newton's second law.

EXAMPLE 1

(a) What is the change in momentum of a body of 500 g mass moving from rest to 3.0 m s^{-1} in 5.0 s?

Solution

$$\begin{aligned}\Delta p &= mv - mu \\ &= 0.500(3.0) - 0 \\ &= 1.5 \text{ kg m s}^{-1}\end{aligned}$$

(b) What is the average resultant force acting on the body?

Solution

$$\begin{aligned}F &= \frac{\Delta p}{\Delta t} \\ &= \frac{1.5}{5.0} \\ &= 0.30 \text{ N}\end{aligned}$$

[1] It is also important to note that the usual definition (or equation $F = ma$) that force is the product of mass and acceleration is no longer accepted as a valid definition for the AS level examination because it is a special situation of the above definition.

EXAMPLE 2

A 100 g ball moving towards a wall at 3.0 m s^{-1} hits the wall and bounces back with the same speed after a brief contact of 0.20 s with the wall. Calculate the average force acting on the ball during its contact with the wall.

Solution

$$F = \frac{\Delta p}{\Delta t}$$
$$= \frac{-0.100(3.0) - 0.100(3.0)}{0.20}$$
$$= 3.0 \text{ N}$$

3.5 Newton's third law

Definition of Newton's third law

> *For every action force acting on a body, there is an equal in magnitude force acting on the other body in the opposite direction.*

For two forces to be considered as the action-reaction pair, they need to satisfy the following conditions:

- equal in magnitude
- opposite direction
- same in nature
- acting on different bodies

Identifying the action-reaction pair of forces

It is important to identify action-reaction pairs of forces. While the weight and the normal contact force acting on a body at rest on a surface are equal and opposite, these two forces are not action-reaction pair.

To identify action-reaction pair, you would need to first determine the action force acting on a body A due to another body B. The reaction force would be the force acting on body B due to body A.

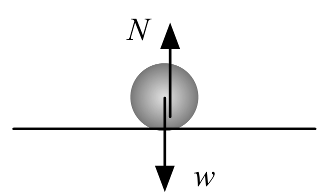
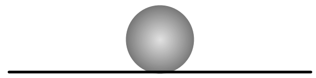


FIG. 3.2 FORCES ACTING ON A BODY AT REST ON A SURFACE

N and *w* are not action-reaction pair even through they are equal and opposite.

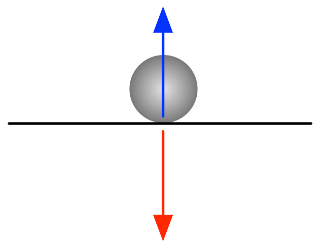
Example 3

Identify the reaction force of the normal contact force acting on a ball resting on a surface.



Solution

It is a good practice to draw the forces on a body. This is called the free body diagram.



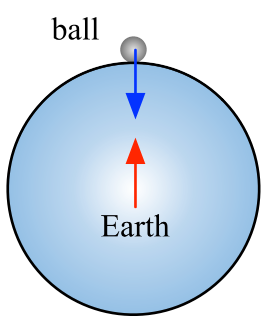
The action force is the normal contact force acting on the ball by the surface.

The reaction force is the normal contact force acting on the surface by the ball.

Example 4

Identify the reaction force of the weight of the ball in Example 3.

Solution



The action force is the force of gravity acting on the ball by the Earth.

The reaction force is the force of gravity acting on the Earth by the ball.

3.6 Weight and its effect on motion

Definition of weight:

> *Weight is the force of gravity on a mass.*

A mass in a gravitational field experiences weight. Since weight is the product of mass and the gravitational field strength, its value is constant.

$$w = mg \tag{3.4}$$

where w is the weight of the object, m is the mass and g is the gravitational field strength.

The resultant force acting on the mass is always towards the ground and has the magnitude of mg. This is regardless of whether the moving is moving up or down (**Fig. 3.1**).

3.7 Falling with air resistance

An object falling in air experiences air resistance. Its acceleration will not be constant as the resultant force on the mass is different as compared to the situation when there is no air resistance.

Object moving up: resultant force $= mg + R$

Object moving down: resultant force $= mg - R$

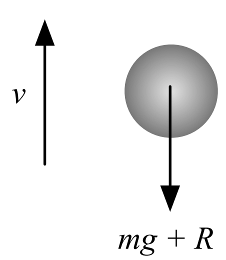


FIG. 3.3A AN OBJECT MOVING UP IN A MEDIUM

Resistance is always in opposite direction to motion. Since it is moving up, the air resistance would be downwards. Thus, the net force is the sum of weight and resistance. This results in a larger deceleration in the upward direction.

$$a = \frac{W + R}{m}$$

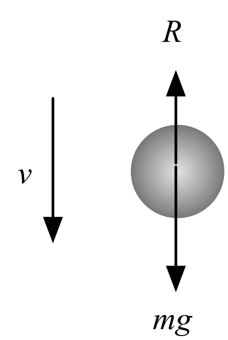


FIG. 3.3B AN OBJECT MOVING DOWN IN A MEDIUM

Since the ball is moving down, the air resistance would be upwards. Thus, the net force is the difference of weight and resistance. This results in a smaller acceleration in the downward direction.

$$a = \frac{W - R}{m}$$

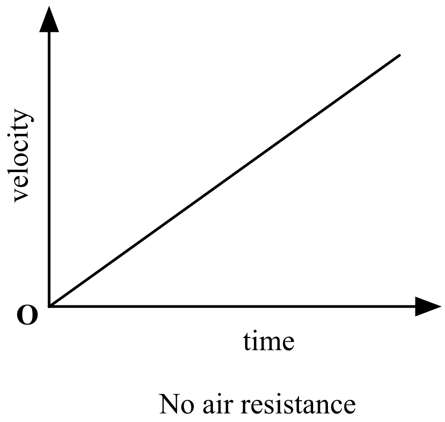


FIG. 3.4A VELOCITY-TIME GRAPH OF AN OBJECT FALLING WITHOUT AIR RESISTANCE

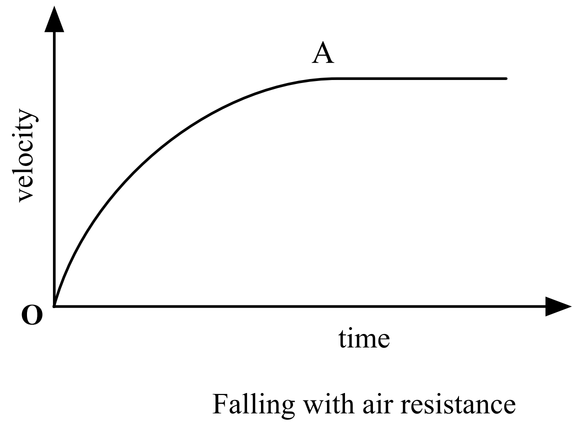


FIG. 3.4B VELOCITY-TIME GRAPH OF AN OBJECT FALLING WITH AIR RESISTANCE

When the object starts falling at O, it falls with an acceleration of g. Thus, the gradient at O has a magnitude of g. As it fall further, the gradient (deceleration) decreases, until it reaches point A from which it will travel at its terminal velocity. The deceleration is zero from A onwards.

3.8 Conservation of momentum

The principle of conservation of momentum states that in the absence of external forces, the total momentum in a system before and after a collision is constant.

This conservation law applies to all situations if the whole system is considered. A system refers to all the interacting bodies such that there are no external forces acting on any of the bodies in the system. In the case of a falling body, it is obvious that its momentum changes. An external force in the form of its weight acts on the falling object and so it is not considered as an isolated system. However, if the cause of the weight, which is the Earth, is considered together as a system, then the law of conservation of momentum still hold. In such a situation, the increase in the body's momentum (when it is falling down) is compensated by the decrease in the Earth's momentum (which is also accelerating towards the body, thus negating any increase in the ball's momentum).

3.9 Collisions

There are in general, two types of collisions in terms of kinetic energy:

Elastic collision

The kinetic energy of the colliding bodies before and after the collision is constant.

In an elastic collision, the relative speed of approach is equal to the relative speed of separation.

relative speed of approach = $u_B - u_A$

relative speed of separation = $v_A - v_B$

$$u_B - u_A = v_A - v_B$$

This formula can be used regardless of weather the masses of A and B are equal or not.

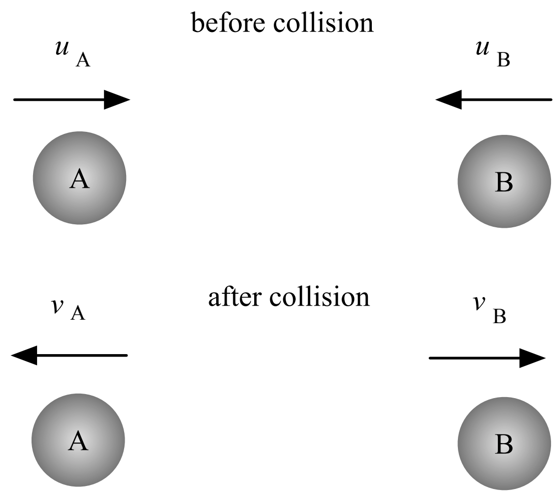
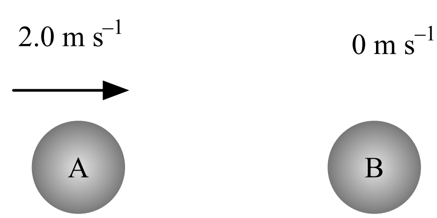


FIG. 3.5 VELOCITIES OF BODIES BEFORE AND AFTER COLLISIONS IN AN ELASTIC COLLISION.

EXAMPLE 5

Ball A is moving towards a stationary ball B with a speed of 2.0 m s^{-1}. If the collision is elastic, calculate the speeds of the two balls after separation. Assume that both balls have the same mass.



Solution

relative speed of approach = relative speed of separation

Assuming that both A and B are moving to the right with speed v_A and v_B after collision,

$$u_A - u_B = v_B - v_A$$
$$2 - 0 = v_B - v_A$$
$$v_B - v_A = 2$$

By conservation of momentum,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$
$$2 + 0 = v_A + v_B$$
$$\therefore v_A = 0, v_B = 2 \text{ m s}^{-1}$$

Inelastic collision

The kinetic energy of the colliding bodies after the collision is less than before the collision. The kinetic energy is lost to other forms of energy, such as elastic potential energy, heat or sound. In a perfectly inelastic collision, the colliding bodies stick together. This scenario represents the greatest loss of kinetic energy in the collision.

However, the total momentum in a collision (elastic and inelastic) is always constant.

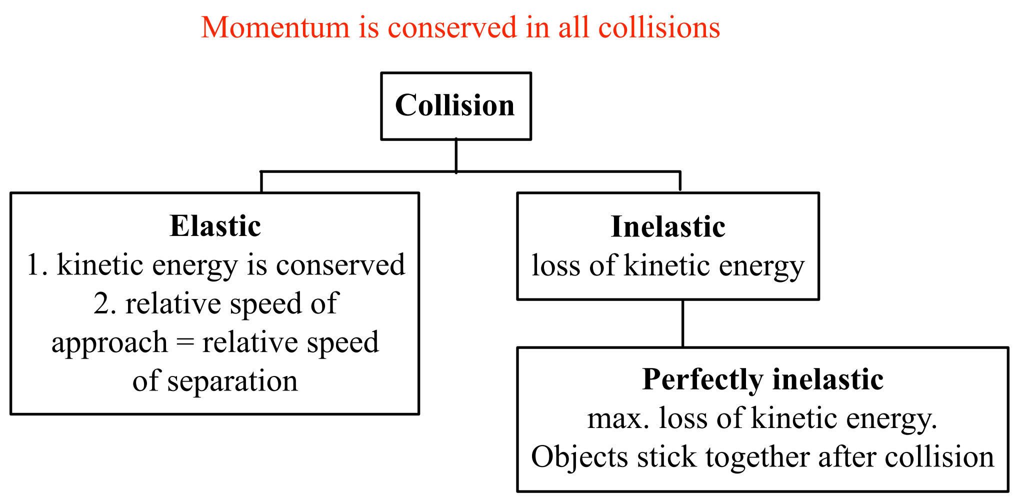


FIG. 3.6 DIAGRAM SUMMARISES THE TWO TYPES OF COLLISIONS: ELASTIC AND INELASTIC COLLISIONS.

End of Chapter

4

Forces

4.1 Turning effects of forces

Moment

The moment of a force equals to the product of the force and the perpendicular distance of the pivot from the force.

$$\text{moment} = F \times d_{\perp} \tag{4.1}$$

where $d_{\perp}$ is the perpendicular distance of the force F from the pivot.

Principle of moment

For any object that is in equilibrium, the sum of the clockwise moments about any point provided by the forces acting on the object equals to the sum of the anticlockwise moment about the same point.

Couple and torque

A couple is a pair of forces that has the following characteristics:

1. equal in magnitude
2. parallel but opposite in direction
3. separated by a distance d

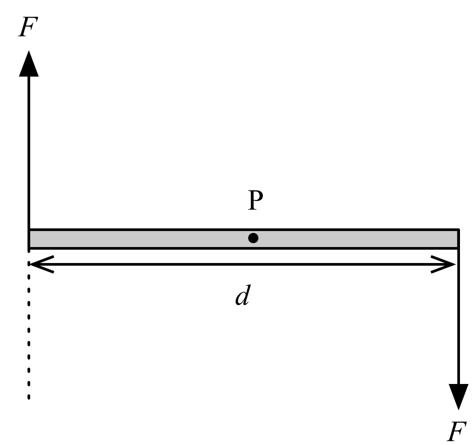


FIG. 4.1 A COUPLE

A couple is a pair of forces that are equal in magnitude, parallel but opposite and separated by a distance.

Torque is the moment produced by a couple.

Torque of a couple = one of the two forces × perpendicular distance between the forces

$$\tau = F \times d_{\perp} \tag{4.2}$$

Note that **Eq (4.2)** does not invalidate the usual way to calculate moment as in **Eq (4.1)**. It can be shown that **Eq (4.1)** and **Eq (4.2)** are consistent.

4.2 Conditions for equilibrium

There are two conditions to be satisfied if a body is in equilibrium. Equilibrium is the condition when an object is neither accelerating in any direction nor is it rotating about any pivot if it was initially not in any rotation motion[2].

1. The net force on the object is zero.
2. The net moment on the object is zero.

If condition 1 is not met, the body would accelerate in the direction of the net force. Similarly, if condition 2 is not met, the body would rotate in the direction of the net moment.

A common scenario of the application of the principle of moment is the case of a uniform rod of weight W held in balance by a rope of tension T as shown in **Fig. 4.2**.

Since the rod is in equilibrium, then it must satisfy both conditions of equilibrium as explained next:

[2] In CIE questions, objects are always in non-rotating setups. As such, equilibrium condition #2 requires that the object to stay in non-rotating motion since it has no initial angular momentum, the rotation equivalent of linear momentum.

Net force is zero

The horizontal component of T equals to the horizontal component of R. The vertical components of T and R equals to the weight W. The object does not accelerate.

$$R\cos\alpha = T\cos\theta$$
$$R\sin\alpha + T\sin\theta = W$$

Net moment is zero

The clockwise moment produced by W equals to the anticlockwise moment produced by T. If the object is originally at rest, it will not start to rotate.

$$W \times \frac{d}{2} = T\sin\theta \times d$$

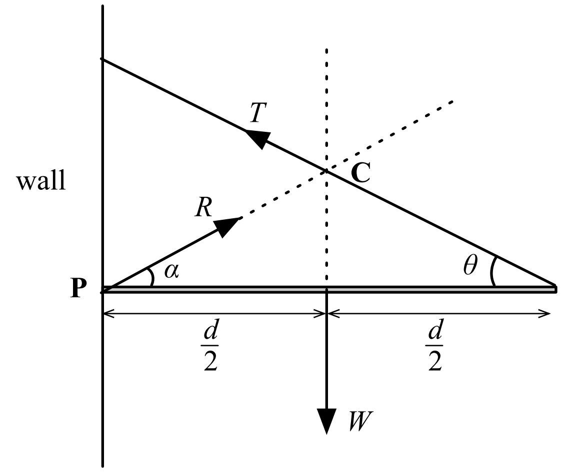


FIG. 4.2 A UNIFORM BEAM IN EQUILIBRIUM HAS NO NET FORCE AND NO NET MOMENT.

It can be seen that the three forces, W, T and R, must pass through the same point at C so that no net moment is created by any of the forces.

End of Chapter

5

Pressure and density

5.1 Density and pressure

Definition of density:

Density is defined as the mass per unit volume.

$$\rho = \frac{M}{V} \tag{5.1}$$

where ρ is the density of the substance, M is the mass of the substance and V is the volume of the substance.

Definition of pressure:

Pressure is defined as the force per unit cross-sectional area.

$$P = \frac{F}{A} \tag{5.2}$$

where F is the force applied on the area A.
The SI unit of pressure is pascal (Pa).

$$1 \text{ Pa} = 1 \text{ N m}^{-2}.$$

5.2 Pressure in a fluid

A fluid can be a gas or a liquid. The pressure of a fluid P increases with depth h.

$$P = h\rho g \tag{5.3}$$

where ρ is the density of the liquid and g is the acceleration due to gravity.

The pressure depends on three factors:

1. depth
2. density of fluid
3. acceleration due to gravity

Deriving the equation for pressure in a liquid

$$\text{volume of water} = A \times h$$
$$\text{mass of water} = \text{density} \times \text{volume} = \rho \times A \times h$$
$$\text{weight of water} = \text{mass} \times g = \rho A h g$$
$$\text{pressure} = \frac{\text{weight}}{\text{area}} = \frac{\rho A h g}{A}$$
$$\therefore P = \rho g h$$

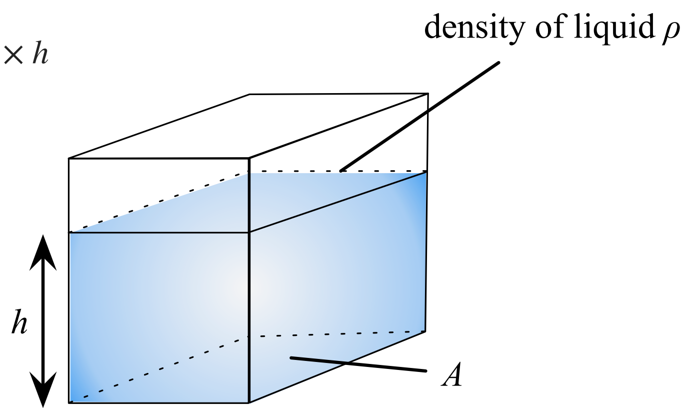


FIG. 5.1 DIAGRAM SHOWS THE DEFINITIONS OF THE PARTS USED TO DERIVE THE PRESSURE OF LIQUID AT A DEPTH OF *h*.

5.3 Kinetic model of matter

The kinetic model of matter attempts to explain the behaviour of matter in terms of the motion of particles and how they are organised with relation to each other.

Assumptions of kinetic theory of matter

- Matter is made up of tiny particles
- The particles tend to move about randomly.
- The particles are moving at different high speeds but the average speed of all the particles is directly proportional to the thermodynamics temperature of the gas.

The kinetic model of matter can explain the cause of pressure in gas as well as deducing the factors that will affect the pressure.

Cause of pressure

- Pressure is caused by the collisions of air molecules with the wall of the container.
- When a molecule collides with a surface, the molecule bounces off.
- The change in momentum of the molecule creates a force on the force.
- The force exerted on the wall due to all the molecules create the pressure.

Factors affecting the pressure

1. the number of molecules that hit each side of the box in one second
2. the force with which one molecule collides with the wall.

End of Chapter

6
Solid deformation

A force acting on a solid in one dimension, can be classified to two types:

- tensile(pulling apart)
- compressive(pushing together)

6.1 Hooke's Law

Definition of Hooke's law:

> *Hooke's law states that, provided the elastic limit is not exceeded, the extension of a body is proportional to the applied load.*

$$F = kx \tag{6.1}$$

where k is the spring constant and x is the extension of the spring.

In a extension-force graph (**Fig. 6.1**), the spring constant may be obtained from the gradient of the graph.

Hooke's Law is only applicable when the extension is not beyond the limit of proportionality. Elastic limit is the point which the spring would not return to the original length after the applied force is removed.

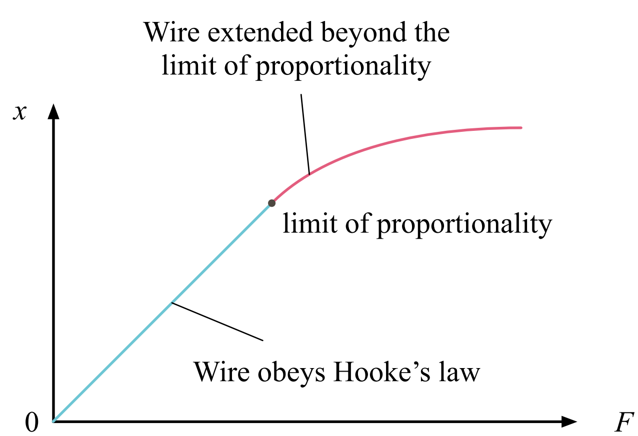


FIG. 6.1 EXTENSION–FORCE GRAPH OF A PULLED WIRE

Hooke's law is only applicable when the limit of proportionality is not exceeded.

6.2 Young Modulus

Definition of stress:

Stress is the tensile or compressive force acting normally to an area.

$$\begin{aligned} \text{stress} &= \frac{\text{force}}{\text{area}} \\ &= \frac{F}{A} \end{aligned} \tag{6.2}$$

Definition of strain:

Strain is the ratio of extension over the original length

$$\begin{aligned} \text{strain} &= \frac{\text{extension}}{\text{original length}} \\ &= \frac{x}{L} \end{aligned} \tag{6.3}$$

Within elastic limit, strain is directly proportional to the stress applied.

Young modulus is the ratio of stress to strain.

$$\begin{aligned} \text{Young modulus} &= \frac{\text{stress}}{\text{strain}} \\ &= \frac{\sigma}{\varepsilon} \end{aligned} \tag{6.4}$$

The Young modulus can be obtained from the gradient of a stress-strain graph.

The Young modulus of a substance is a constant. It is independent of the physical form of the substance.

Determining the Young modulus of a metal

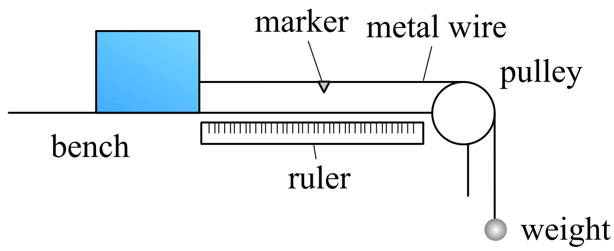


FIG. 6.2 EXPERIMENT TO MEASURE THE YOUNG MODULUS OF A WIRE

- A long wire is used so that the extension can be measured with less uncertainty.
- A travelling microscope is used to read the position of the marker.
- The extension can be obtained by subtracting the new position of the marker from the old position.
- The diameter of the wire is measured using a micrometer screw gauge across the length of the wire, and an average is obtained.
- Once the wire is loaded in increasing steps, the load must be gradually decreased to ensure that here has been no permanent deformation of the wire.
- A graph of stress vs strain is drawn, and the Young modulus is obtained from the gradient of the graph.

6.3 Elastic limit

When the extension of a material falls within the elastic limit, it obeys Hooke's law and extends proportionally to the force applied. When the force is removed, it will return back to the original length. The material is said to be **elastic**.

However, if the extension exceeds the elastic limit, the material will not return back to the original length. The material is said to undergo **plastic deformation**.

6.4 Work done in extension

Energy is expended to extend or compressed a material. The amount of work done can be found from the area under the force-extension graph (**Fig. 6.3A**).

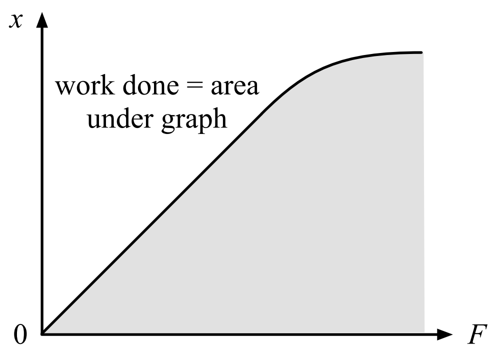


Fig. 6.3A Force-extension graph of a material that is stretched beyond the elastic limit.

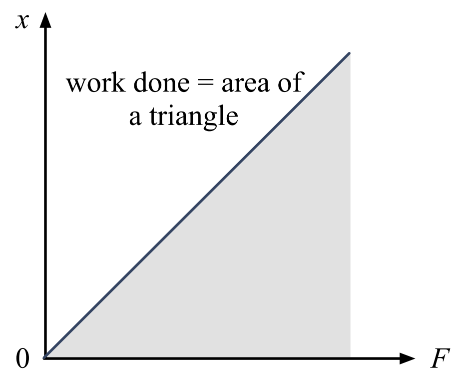


Fig. 6.3B Force-extension graph of a material that is NOT stretched beyond the elastic limit. It obeys Hooke's law.

If the extension of the material does not exceed the elastic limit (**Fig. 6.3B**), then the work done will be the area of a triangle i.e.

$$\text{Work done} = \frac{1}{2}Fx \tag{6.5}$$

Given that the extension is less than the limit of proportionality,

$$\begin{aligned} F &= kx \\ \therefore W &= \frac{1}{2}Fx \\ &= \frac{1}{2}kx^2 \end{aligned} \tag{6.6}$$

End of Chapter

7

Work, energy and power

7.1 Work

Definition of work:

> *Work done by a force is the product of the forces and the distance moved in the direction of the force.*

$$W = F \times s\cos\theta \tag{7.1}$$

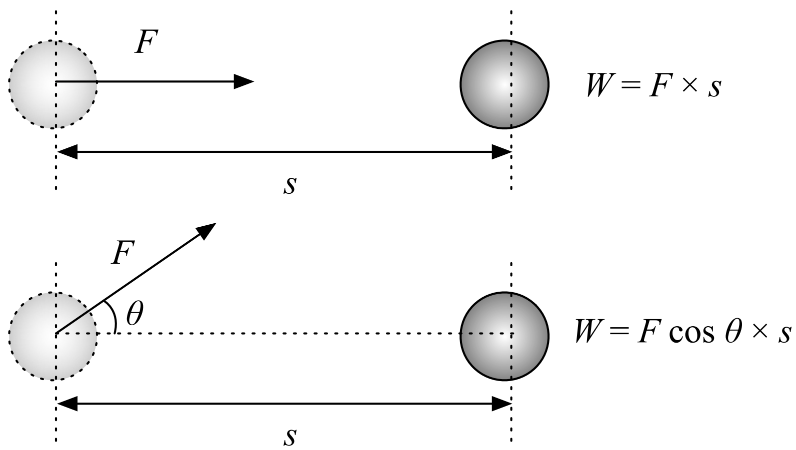


FIG. 7.1 CALCULATION OF WORK DONE

Work done must be calculated using the component of the force in the direction of the displacement.

where W is work done, F is the force acting on the body and s is the displacement the body moves and θ is the direction of s with respect to the force F.

SI unit of energy: joule (J)

1 J = 1 N m

Work done must be calculated by using the component of the force that is parallel to the direction of the displacement.

If the two directions are the same, work is done on the object and the objects gains energy.
If the two directions are opposite, then energy is lost through work done(typically as friction or air resistance)

Work done by a gas

Work done by a gas equals to the product of the pressure it exerts over the cross-sectional area and the displacement the piston moves.

$$\begin{aligned} F &= p \times A \\ W &= p \times A \times s \\ &= p \times V \end{aligned} \tag{7.2}$$

7.2 Efficiency

Definition of efficiency:

> *Efficiency is the ratio of the useful output energy to the total input energy expressed as a percentage.*

$$\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100\,\% \tag{7.3}$$

It is important for you to determine what makes the useful output energy.

Examples of useful output energy in energy conversions

Table 7.1 shows some examples of energy being used to do work and the corresponding energy that are lost or not tapped. The total energy used for efficiency calculation is the sum of both useful energy and the wasted and untapped energies.

Scenario	Useful energy	Wasted energy / energy not tapped
A luggage being delivered from the ground floor to the first floor by an elevator	Gain in gravitational potential energy	• Heat
A man cranking a generator to produce electricity	Electrical energy	• Heat
Water flowing through a hydroelectric damp	Electrical energy	• kinetic energy of water flowing through the damp • heat
Wind flowing through a windmill	Electrical energy	• Kinetic energy of wind that flows through the wind mill • Heat

TABLE 7.1 SHOWS SOME EXAMPLES OF ENERGY BEING USED TO DO USEFUL WORK AND ENERGY THAT ARE EITHER WASTED OR NOT TAPPED. TOTAL ENERGY IS THE SUM OF ALL THESE ENERGIES.

Deriving the formula of gravitational potential energy

In a uniform gravitational field, the weight of the body being raised is a constant, mg.

To raise the body through a height h, a force by an external agent equals to the weight is needed. This force exerted over the distance h, is the work done on the body.

Since the gravitational potential energy equals to the work done by this external agent in raising the height of the body, we can equate the increase in gravitational potential energy to the work done

$$\Delta E_P = mg\Delta h \qquad (7.4)$$

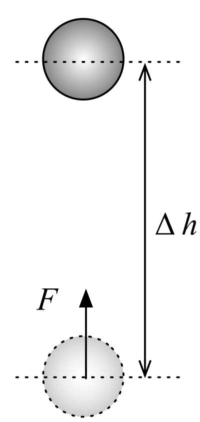


FIG. 7.2 A BALL MOVED AGAINST GRAVITY THROUGH A HEIGHT **Δ*h*** IN THE DIRECTION OF FORCE ***F***.

7.3 Different forms of potential energy

- *Electrical potential energy*: capacitor. A capacitor stores electrical charges of equal but opposite signs on each of its plates. As such, a capacitor does not store any net charge. However, since the charges are separated, it stores electrical potential energy.
- *Elastic potential energy*: spring. A spring stores elastic potential energy when it is stretched or compressed.
- *Chemical potential energy*: food and battery.
- *Nuclear energy*: heavy nuclei releases nuclear energy in the form of kinetic energy of the daughter particles in a radioactive decay.

7.4 Kinetic Energy

Definition of kinetic energy:

> *Kinetic energy is the energy due to motion*

$$E_{\mathrm{K}} = \tfrac{1}{2}mv^2 \tag{7.5}$$

Deriving the formula

An object acted upon by a constant force F accelerates from rest to a velocity v follows the equation

$$v^2 = 2as$$

$$s = \frac{v^2}{2a}$$

Since work = force × distance,

$$\text{work done} = f \times s$$

$$= ma\tfrac{v^2}{2a}$$

$$\therefore E_{\mathrm{K}} = \tfrac{1}{2}mv^2$$

7.5 Gravitational potential energy and kinetic energy transformation

When a high object falls to a lower height, the GPE is converted to KE. The increase in KE cause the object to increase in speed. The decrease in height should always be calculated based on the drop in vertical height.

In **Fig. 7.3**, the gravitational potential energy at point A is converted to kinetic energy at point B. At any point between A and B, the pendulum has a combination of GPE and KE.

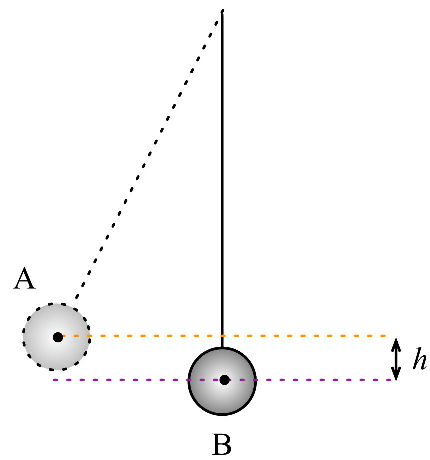


FIG. 7.3 CONVERSION BETWEEN GRAVITATIONAL POTENTIAL ENERGY AND KINETIC ENERGY

Note that the change in height is measured from the centre of mass of the ball.

7.6 Power

Definition of power:

> *Power is defined as the work done per unit time.*

From the definition,

$$P = \frac{W}{\Delta t} \tag{7.6}$$

Since $W = F \times \Delta s$,

$$\begin{aligned} P &= \frac{F\Delta s}{\Delta t} \\ &= F \times \frac{\Delta s}{\Delta t} \\ \therefore P &= Fv \end{aligned} \tag{7.7}$$

The equation means that for an object moving at a constant velocity v due to an applied force F, the power required to maintain the motion is Fv. It is important to understand that this formula can only be applied in situations where F and v is constant.

End of Chapter

About the author

Stanley Sim teaches Physics at high schools for 10 years. He is currently teaching Physics and is the director at Classroom Technologies LLP.

Stanley is an advocate of using technology for learning. He is an Apple Distinguished Educator and uses Mac since the days of Mac OS 9.

In his free time, Stanley loves to code in Ruby on Rails and Swift. He has also experiences in teaching C, C++ and PHP. He created the result management system in his school that uses a database engine and has trained a team of administrators to operate it.

Stanley's goal in life is to create and run a successful company and to spend his free time with his family.

Distinguished Educator

Classroom Technologies